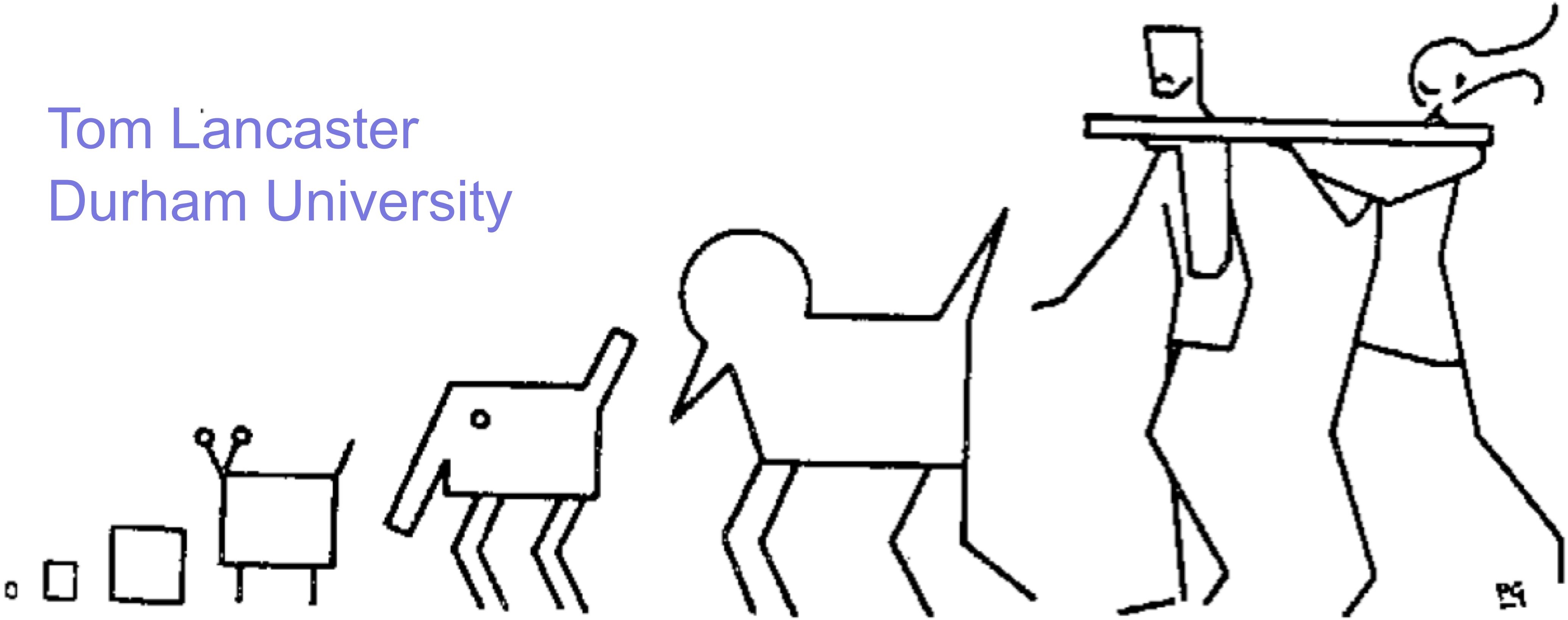
Lecture 1
Muons and (static) magnetism

Tom Lancaster
Durham University



Various animals attempting to follow a scaling law.

Lecture 1:

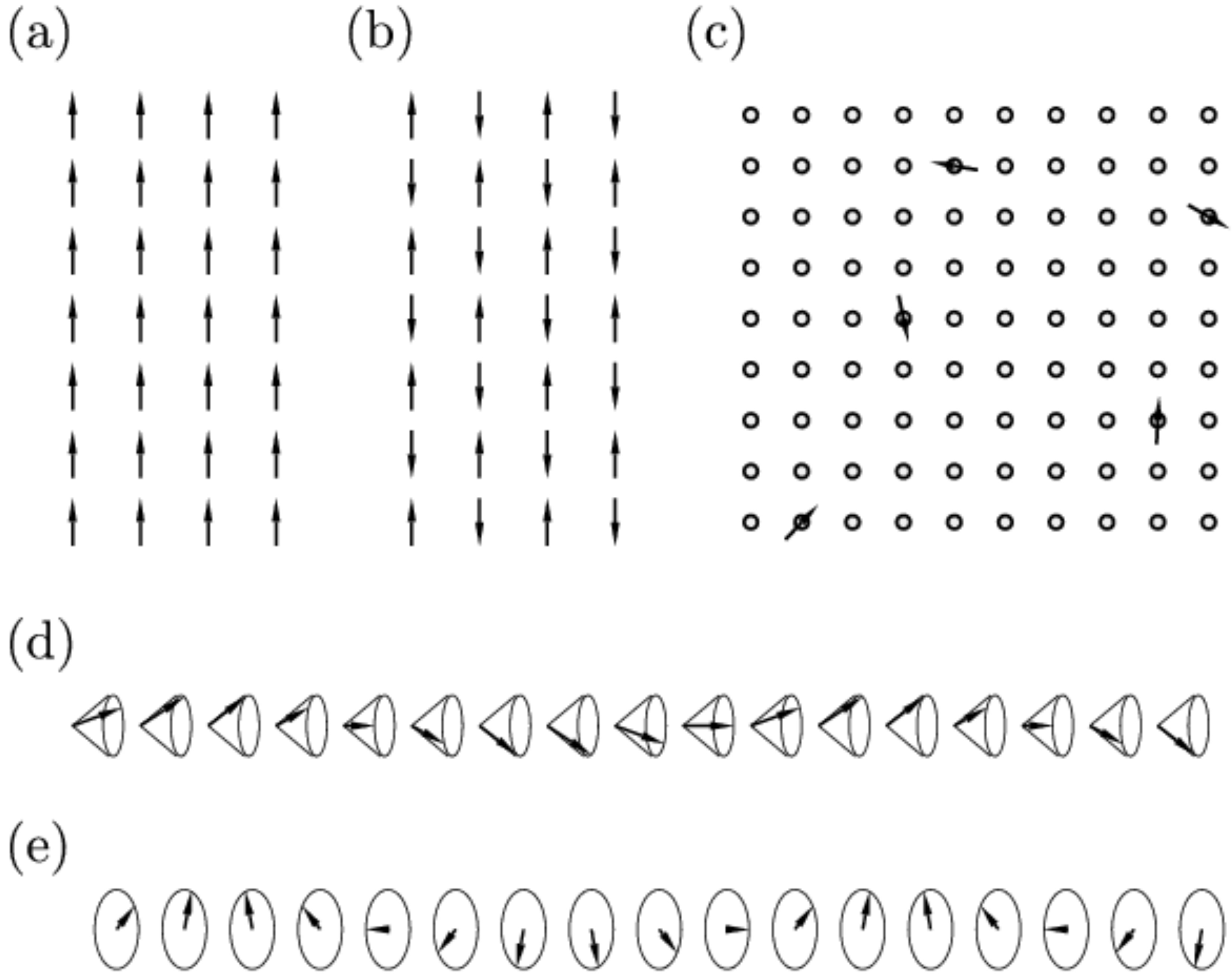
Magnetic order and muon oscillations

Magnetic fields and the muon

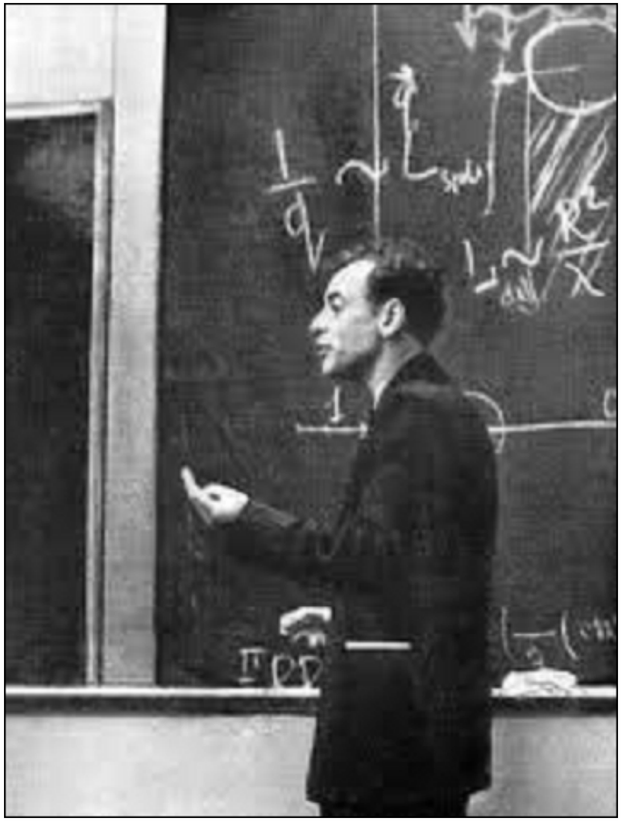
Distributions of fields and the spin density wave

With grateful thanks to S.J. Blundell for permission to use many of his slides!

The many faces of magnetism



How do we understand the occurrence of magnetic order?

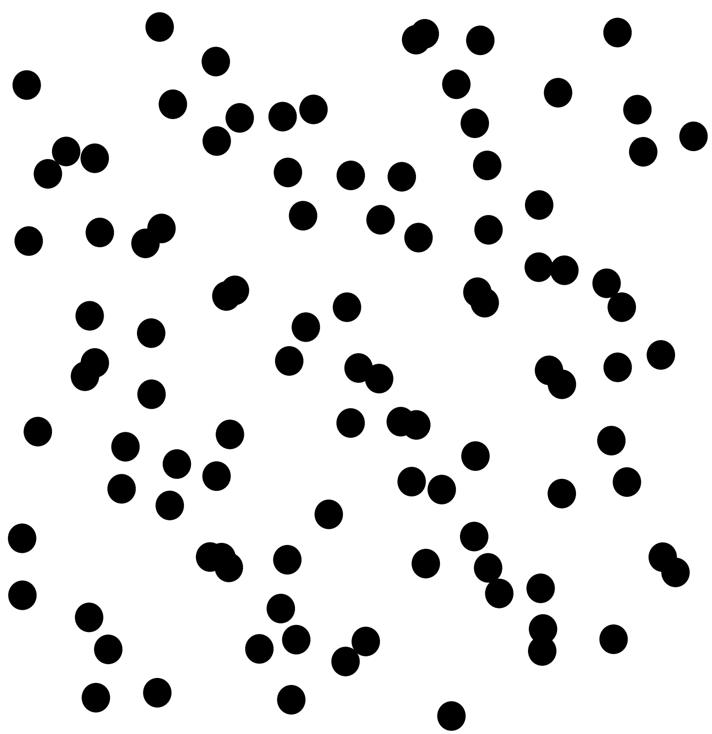


Lev Landau (1908-1968)

Philip Anderson (1923-)

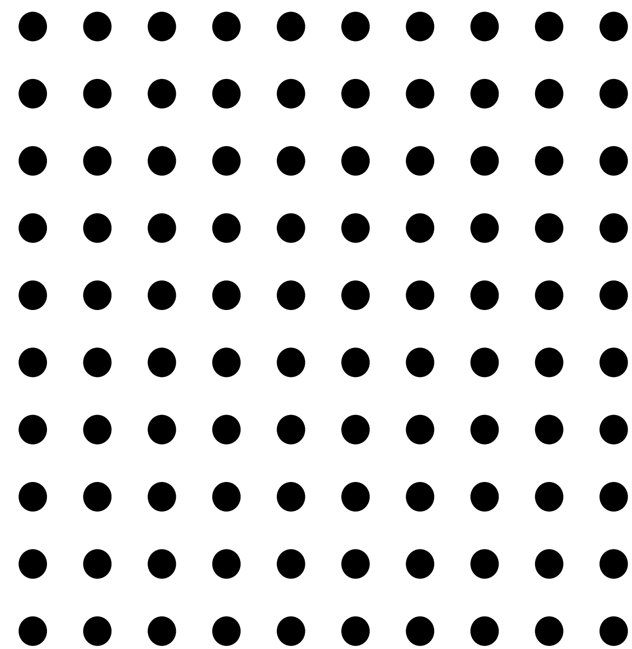
Examples of broken symmetry

Liquid



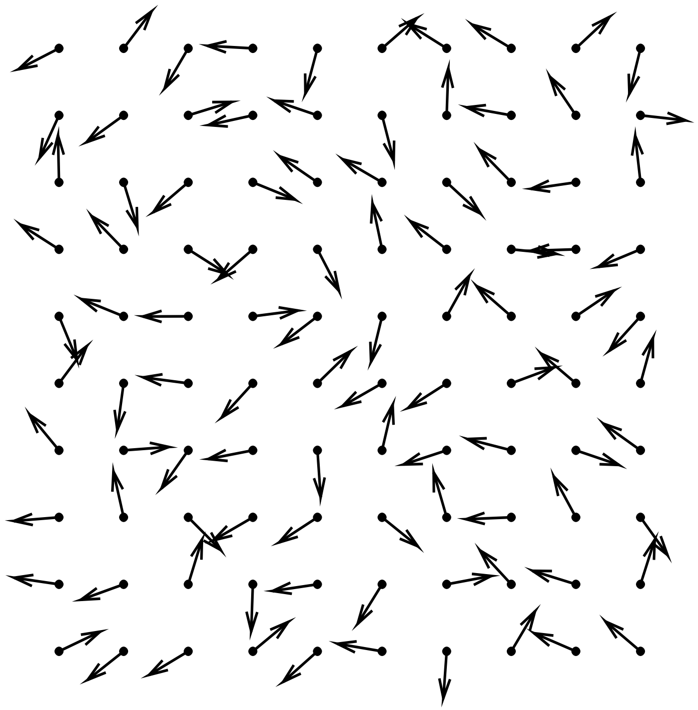
$T>T_c$

Solid



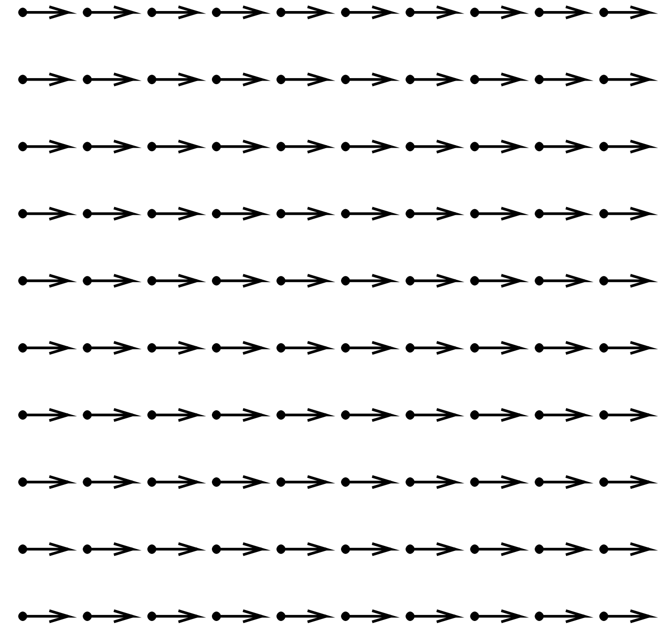
$T<T_c$

Paramagnet



$T > T_c$

Ferromagnet

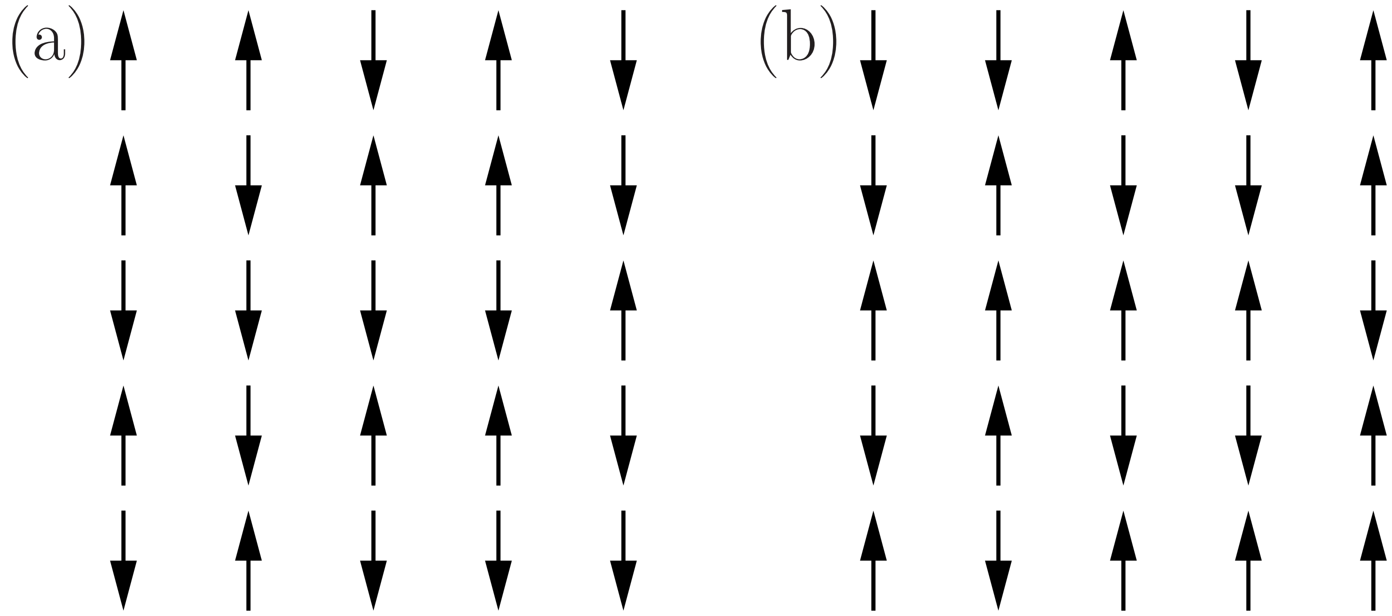


$T < T_c$

Broken symmetry is a cornerstone of CMP

Consider a magnet

$T>T_c$

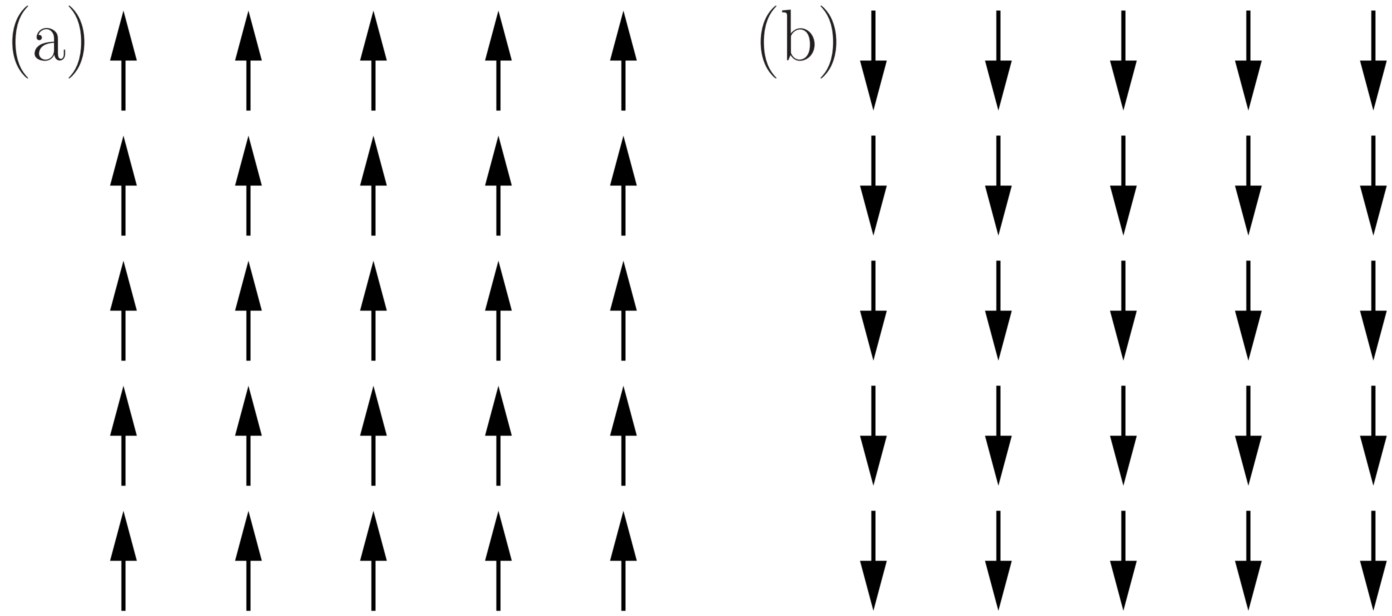


These magnets are the same

Broken symmetry is a cornerstone of CMP

Consider a magnet

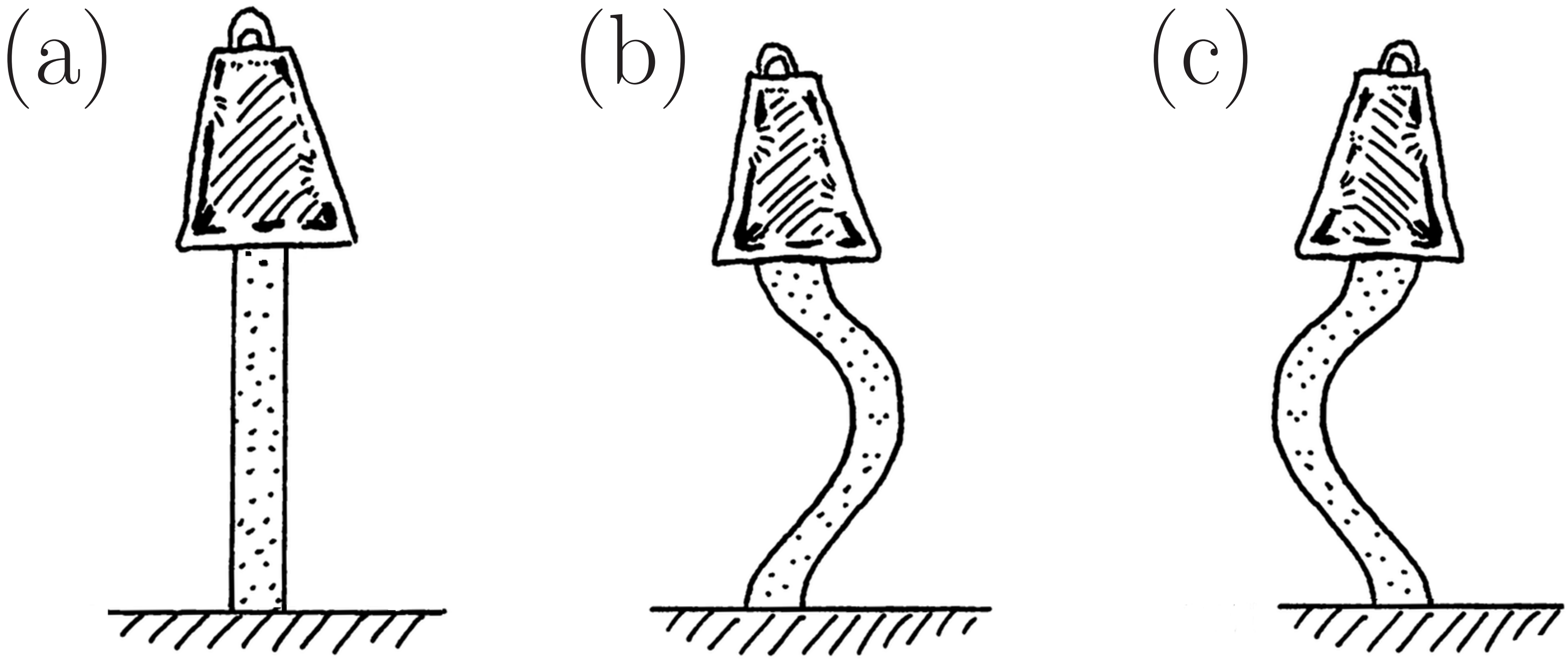
$T < T_c$



These magnets are different

Buridan’s donkey

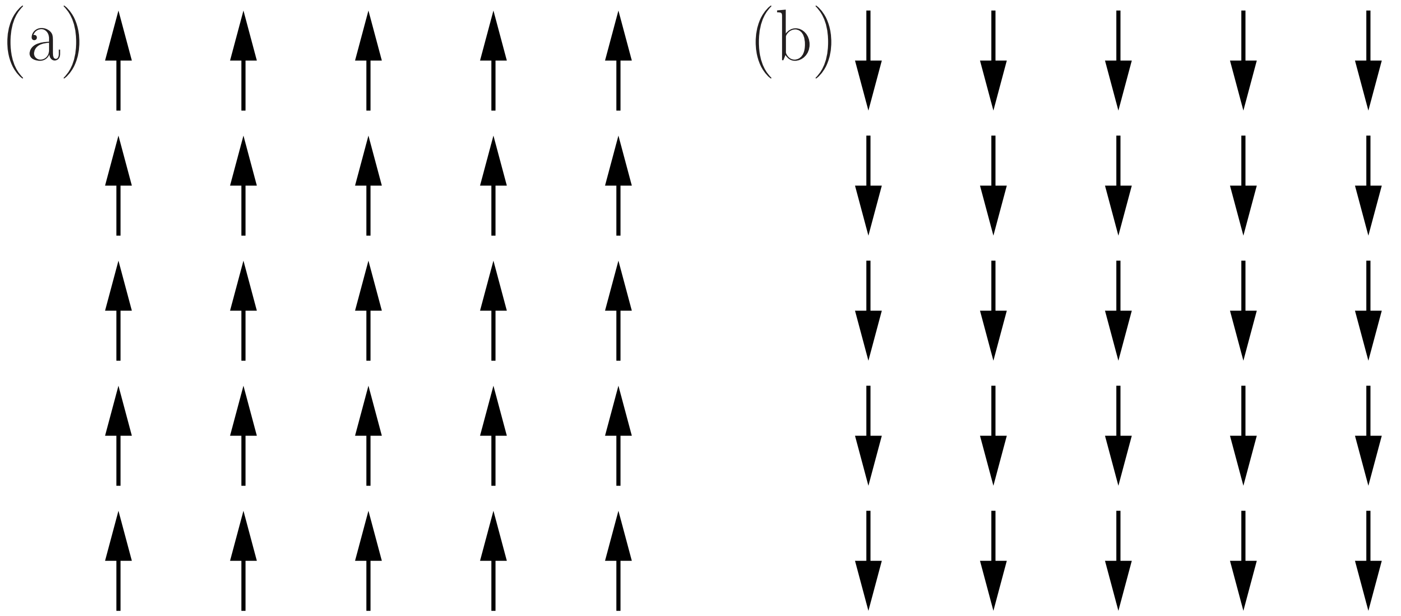
Euler strut



Broken symmetry is a cornerstone of CMP

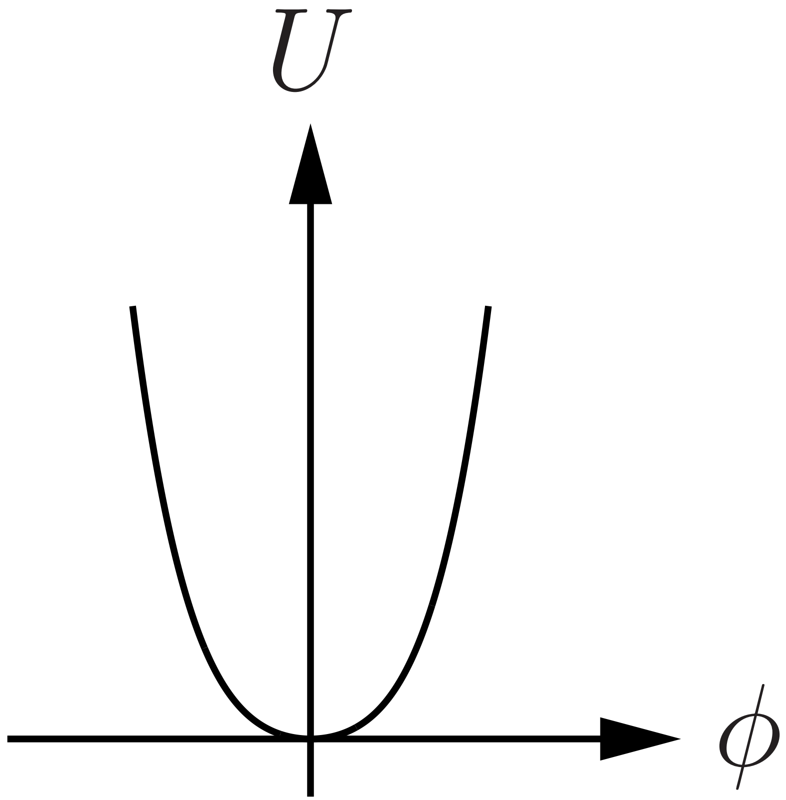
Consider a magnet

$T<T_c$

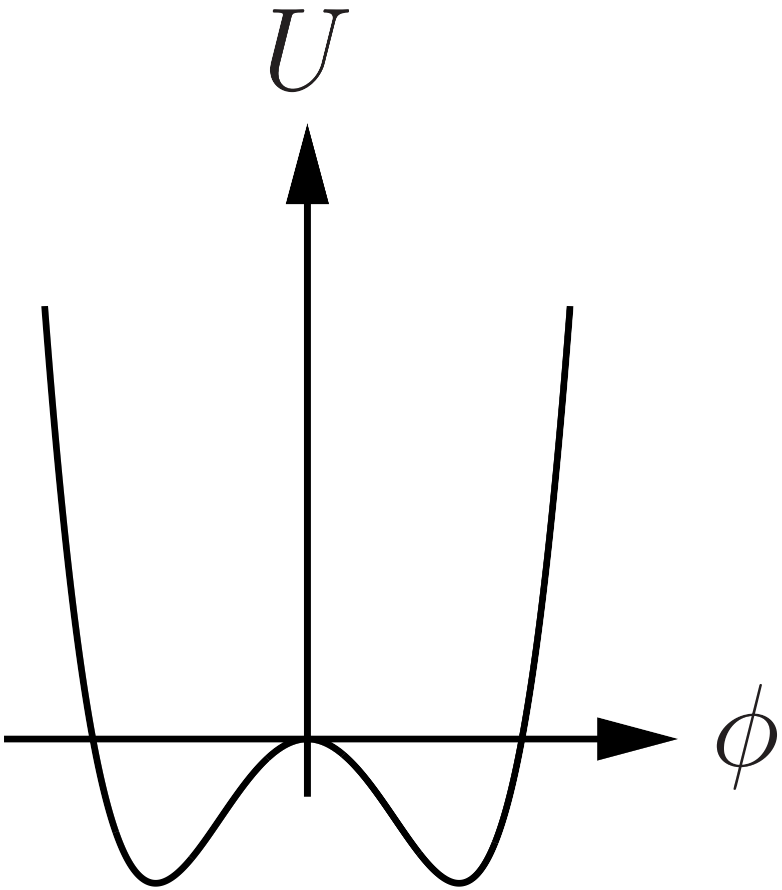


These magnets are different

This has a simple mathematical description

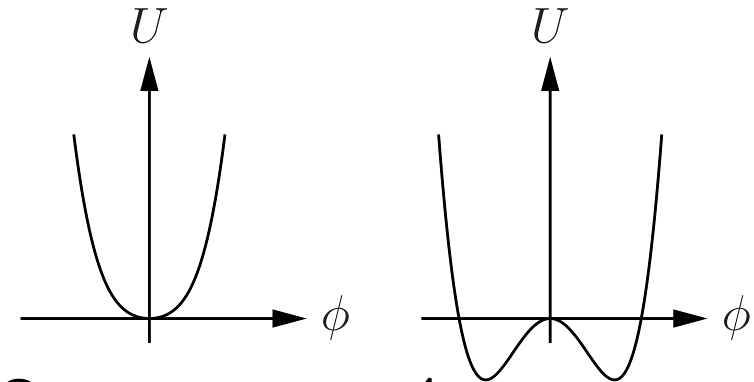


$T > T_c$



$T < T_c$

Landau mean-field theory



$$F = F_0 + a(T - T_\mathrm{c})M^2 + bM^4 + \ldots$$

$$\frac{\partial F}{\partial M} = 2a(T - T_\mathrm{c})M + 4bM^3 = 0$$

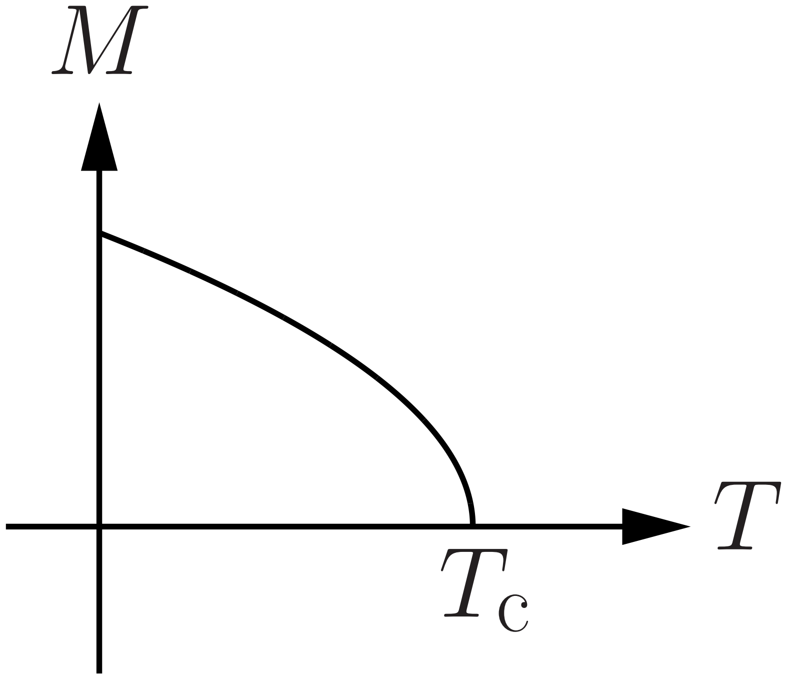
$$M_0^2 = -\frac{a(T - T_\mathrm{c})}{2b}$$

Minima in the free energy may be identified

$$M_0 = \left[\frac{a(T_\mathrm{c} - T)}{2b}\right]^{\frac{1}{2}} \qquad T < T_\mathrm{c}$$

$$= 0 \qquad T > T_\mathrm{c}$$

- Phase transitions



The 4-fold way of broken symmetry

- Phase transitions

Mathematical singularity at T_c

- Rigidity

order transmits forces

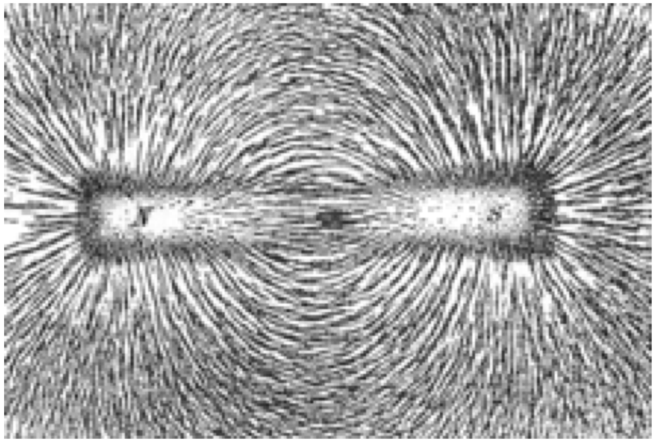
- New excitations

New particle spectrum

- Defects

Walls that separate different order in different places

The magnet

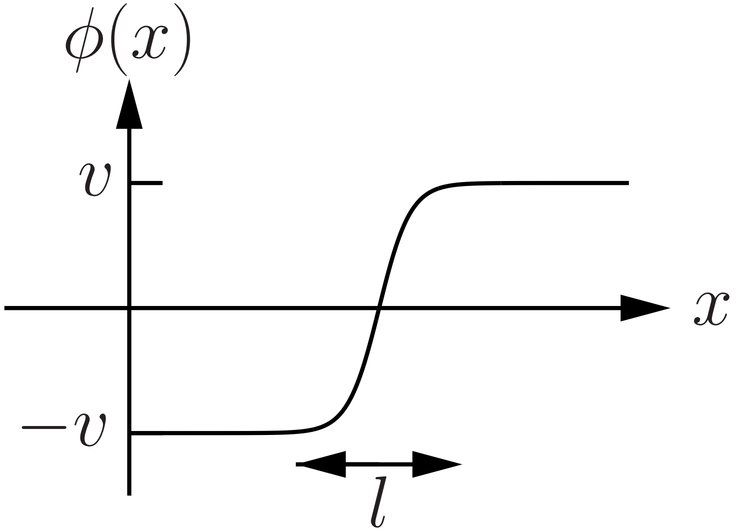


- Order parameter *M*

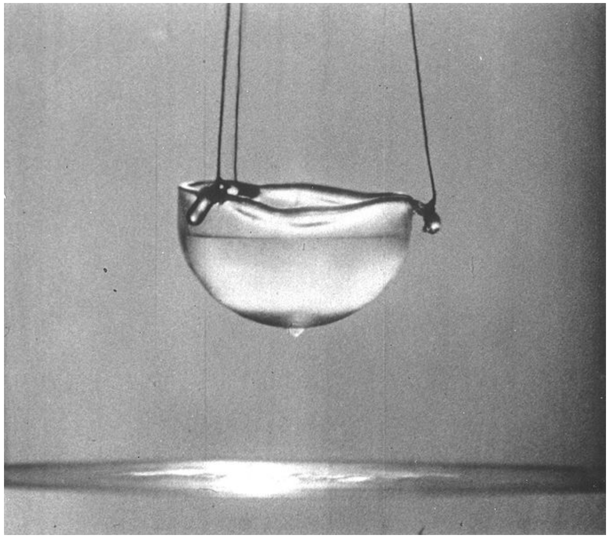
- Rigidity: permanent magnetism

- Excitations: magnon particles

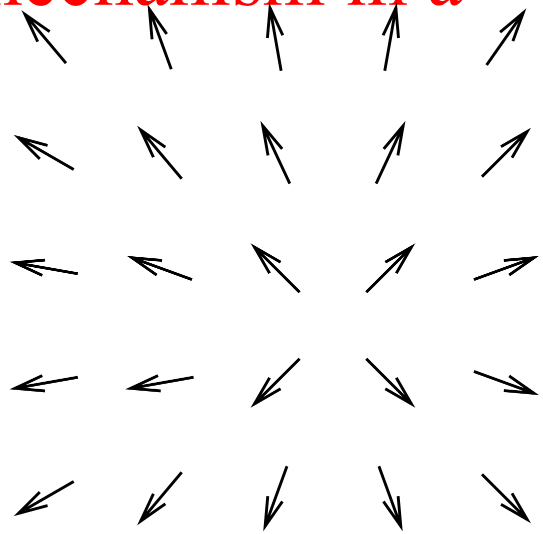
- Defects: domain walls



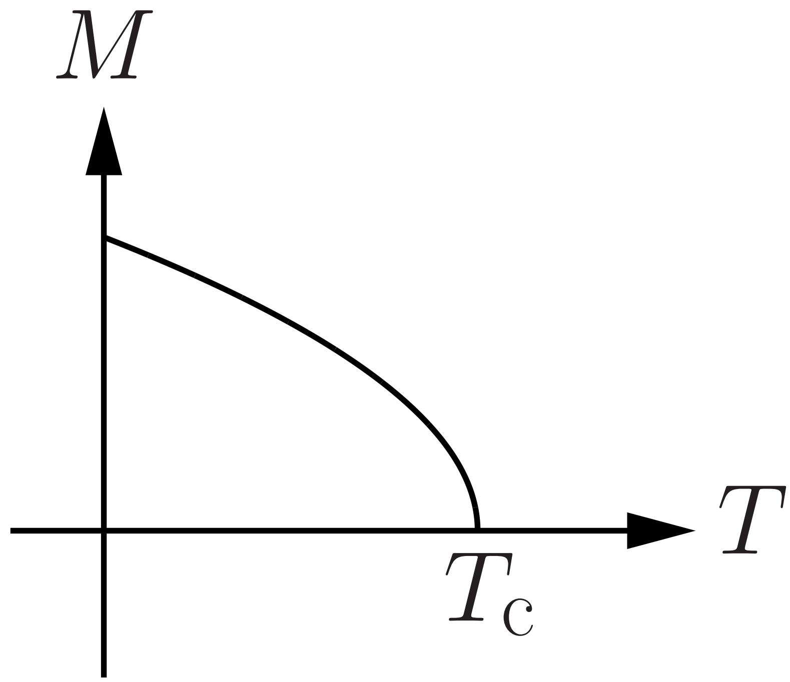
Superfluids and superconductors



- Order parameter: $<\Psi(x)>$
- Rigidity: the supercurrent
- Excitations: Bogolons (via the Higgs mechanism in a SC)
- Defects: vortices



- Phase transitions



Critical exponents

Three commonly discussed

$$
\begin{aligned}
M &\propto (T_\mathrm{c} - T)^\beta & T < T_\mathrm{c} \\
M &\propto H^{1/\delta} & T = T_\mathrm{c} \\
\chi = \frac{\partial M}{\partial H}\Big|_{H\to 0} &\propto |T - T_\mathrm{c}|^{-\gamma} & T \to T_\mathrm{c}
\end{aligned}
$$

Critical exponents

- Heat capacity: $C \sim |t|^{-\alpha}$,
- Magnetization: $M \sim (-t)^{\beta}$, for $B \to 0$, $T < T_{\mathrm{c}}$,
- Magnetic susceptibility: $\chi \sim |t|^{-\gamma}$,
- Field dependence of χ at $T = T_{\mathrm{c}}$: $\chi \sim |B|^{1/\delta}$,
- Correlation length: $\xi \sim |t|^{-\nu}$,
- The correlation function $G(r)$ behaves like

$$G(r) \sim \left\{ \begin{array}{cc} \frac{1}{|r|^{d-2+\eta}} & |r| \ll \xi \\ \mathrm{e}^{-\frac{|r|}{\xi}} & |r| \gg \xi, \end{array} \right\}$$

where r is distance and d is the dimensionality of the system.

Exponents don't rely on any length scale in the system

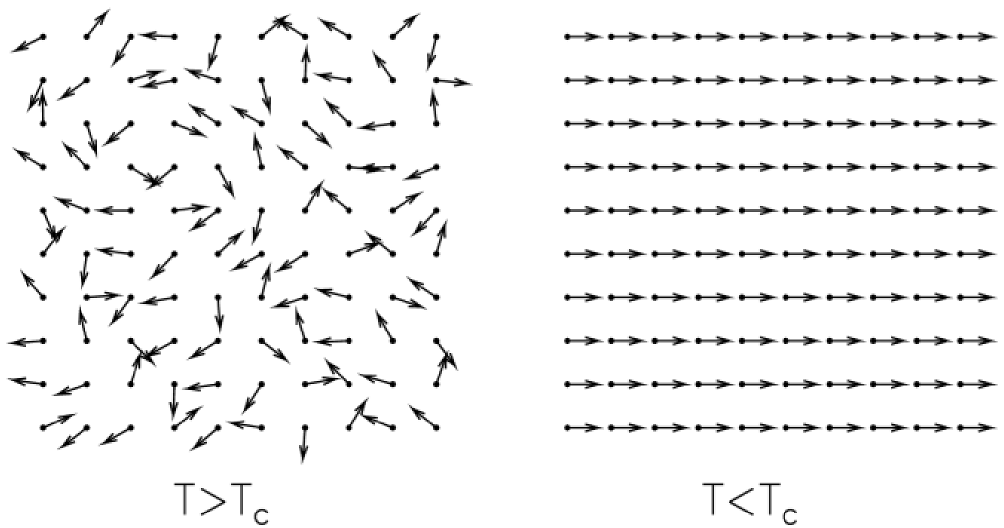
Critical exponents for mean field theory

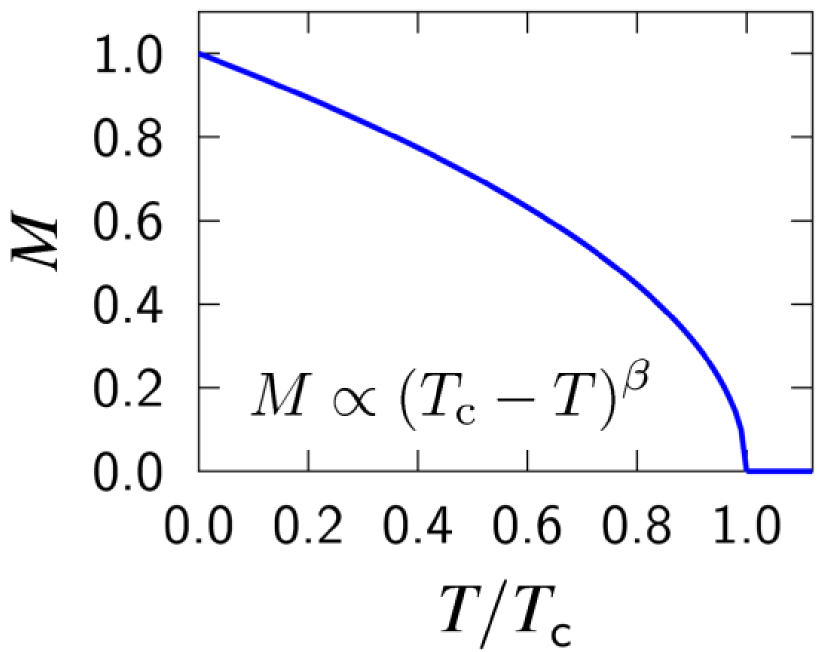
	α	β	γ	δ	ν	η
$\epsilon = 0$ ($d = 4$, mean field)	0	$\frac{1}{2}$	1	3	$\frac{1}{2}$	0

$$\mu$$

The muon

Critical phenomena in magnetism





The parameter β is a *critical exponent*

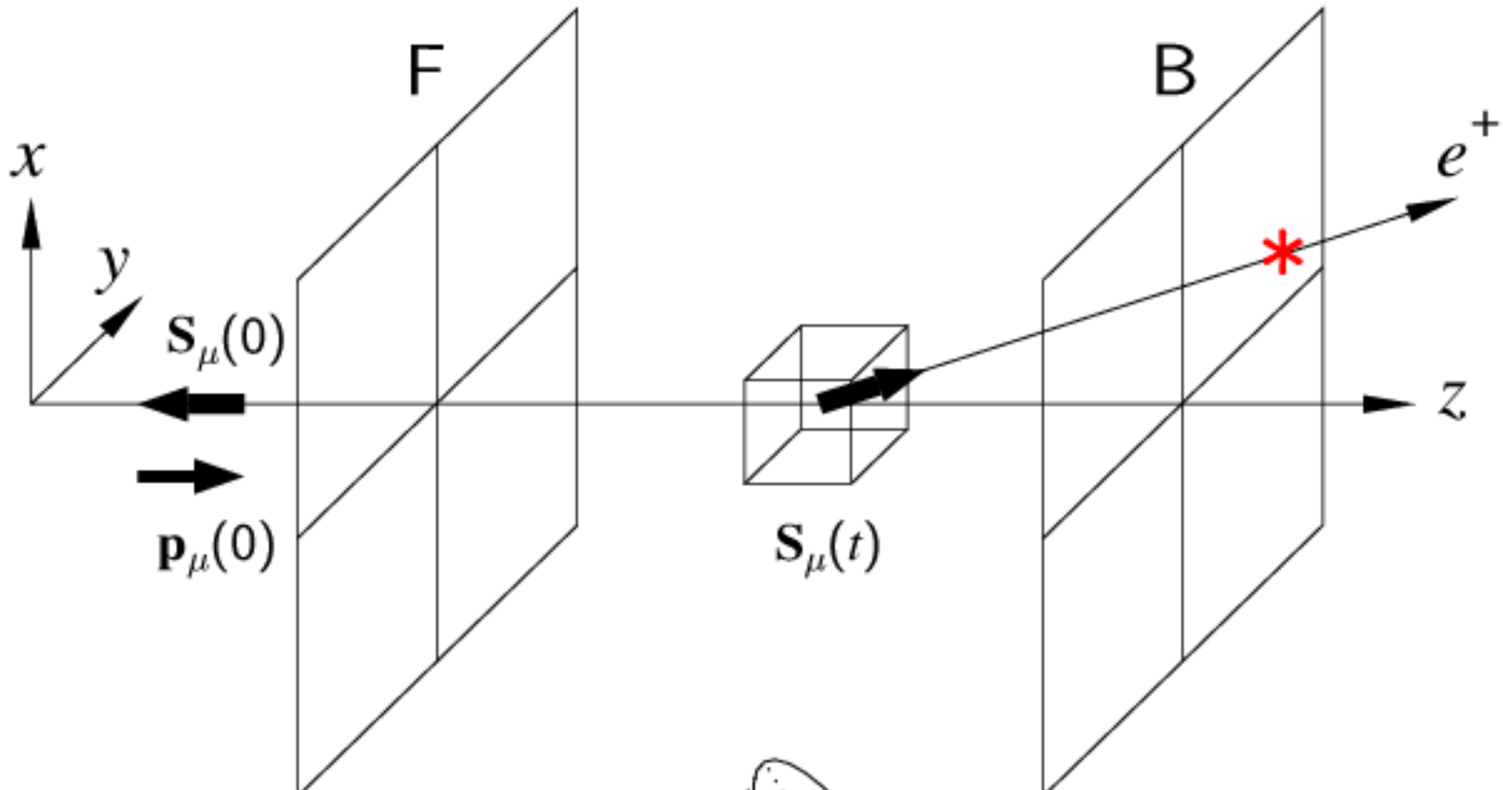
In μ^+SR we measure $\nu(T) \propto M(T)$

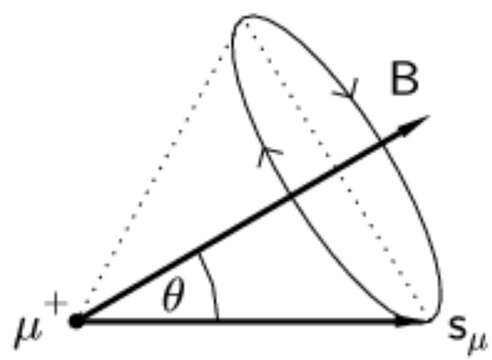
S.J. Blundell *Magnetism in Condensed Matter*

Particle properties

	charge	spin	mass	moment	$\gamma\ /\ 2\pi$ (kHz G^{-1})	lifetime (μs)
e	$\pm$e	1/2	m_e $= 0.51$ MeV	657 μ_p	2800	∞
μ	$\pm$e	1/2	207 m_e $= 105.7$ MeV	3.18 μ_p	13.5	2.19
p	$\pm$e	1/2	1836 m_e $= 938$ MeV	μ_p	4.26	∞

Muon spin relaxation



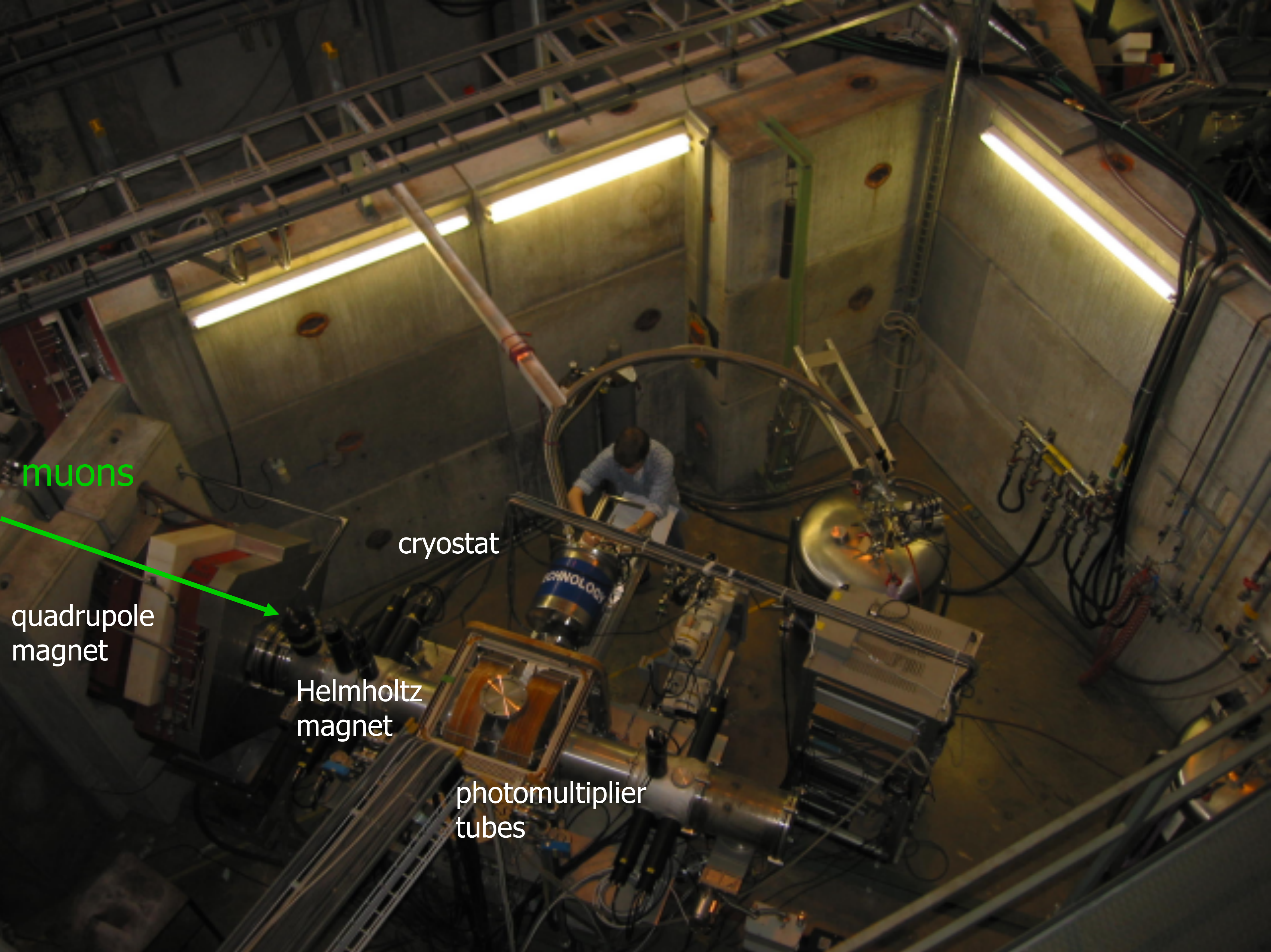


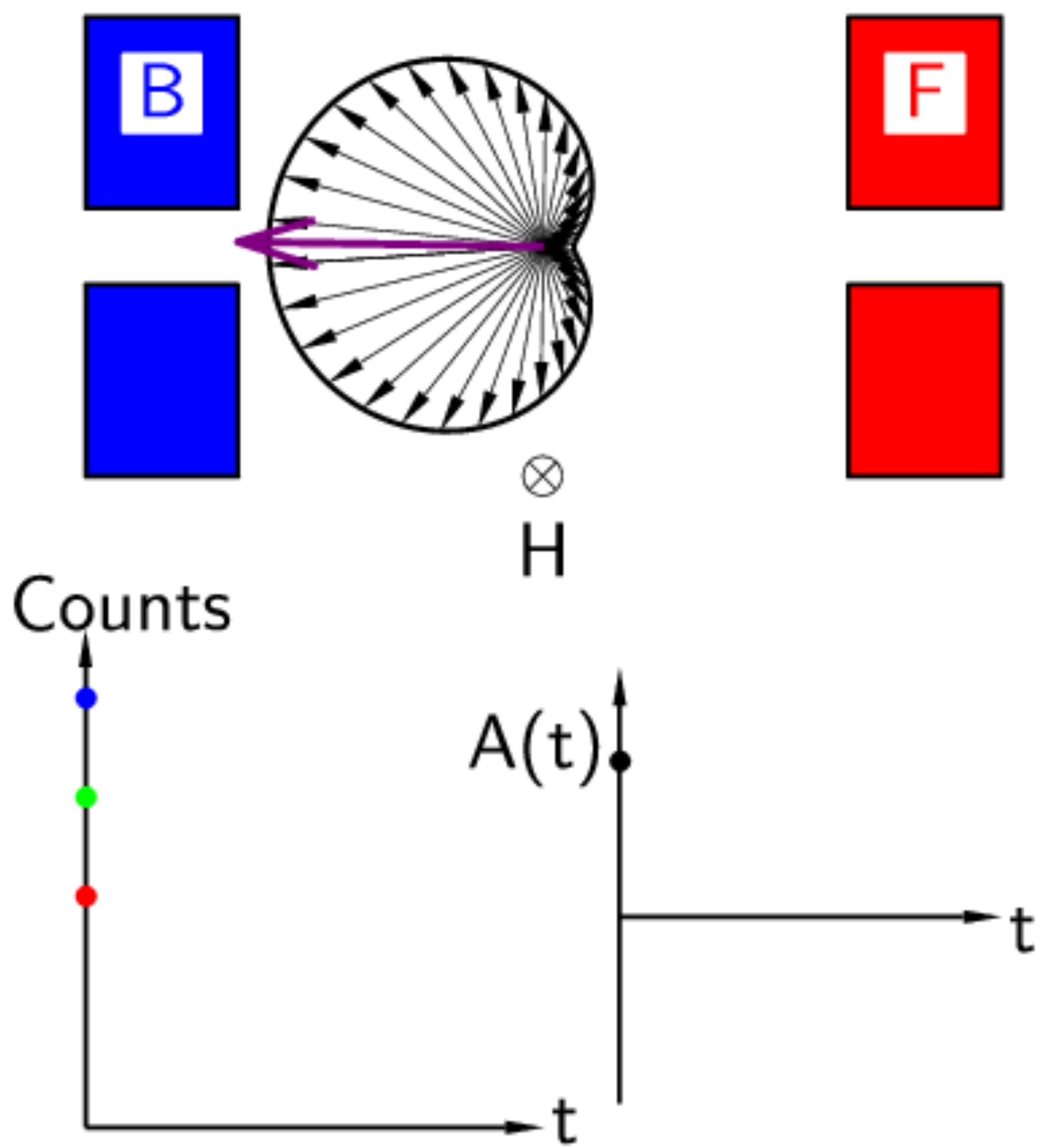
$$A(t) = \frac{N_\mathrm{F} - N_\mathrm{B}}{N_\mathrm{F} + N_\mathrm{B}}$$

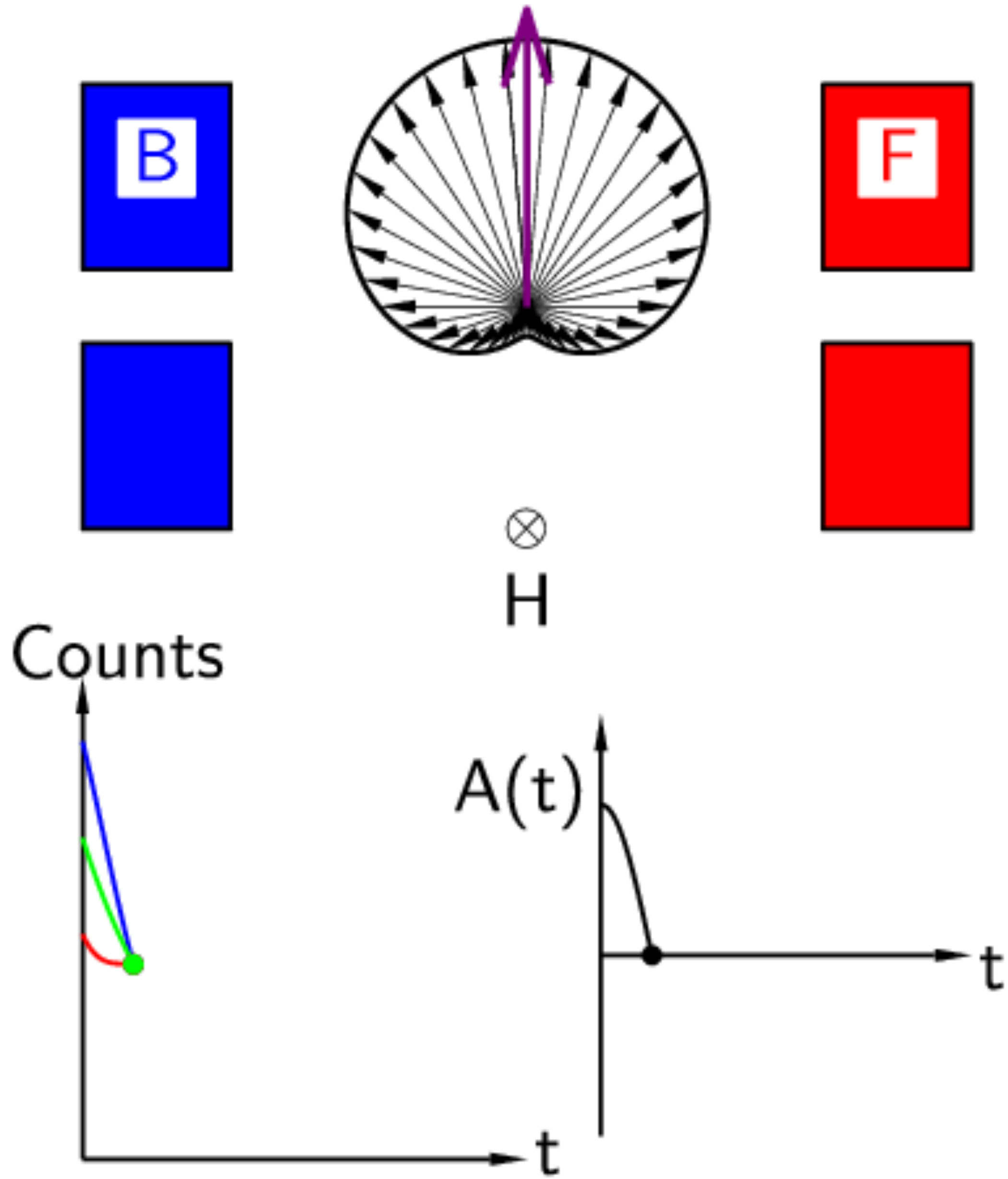
spin antiparallel to momentum

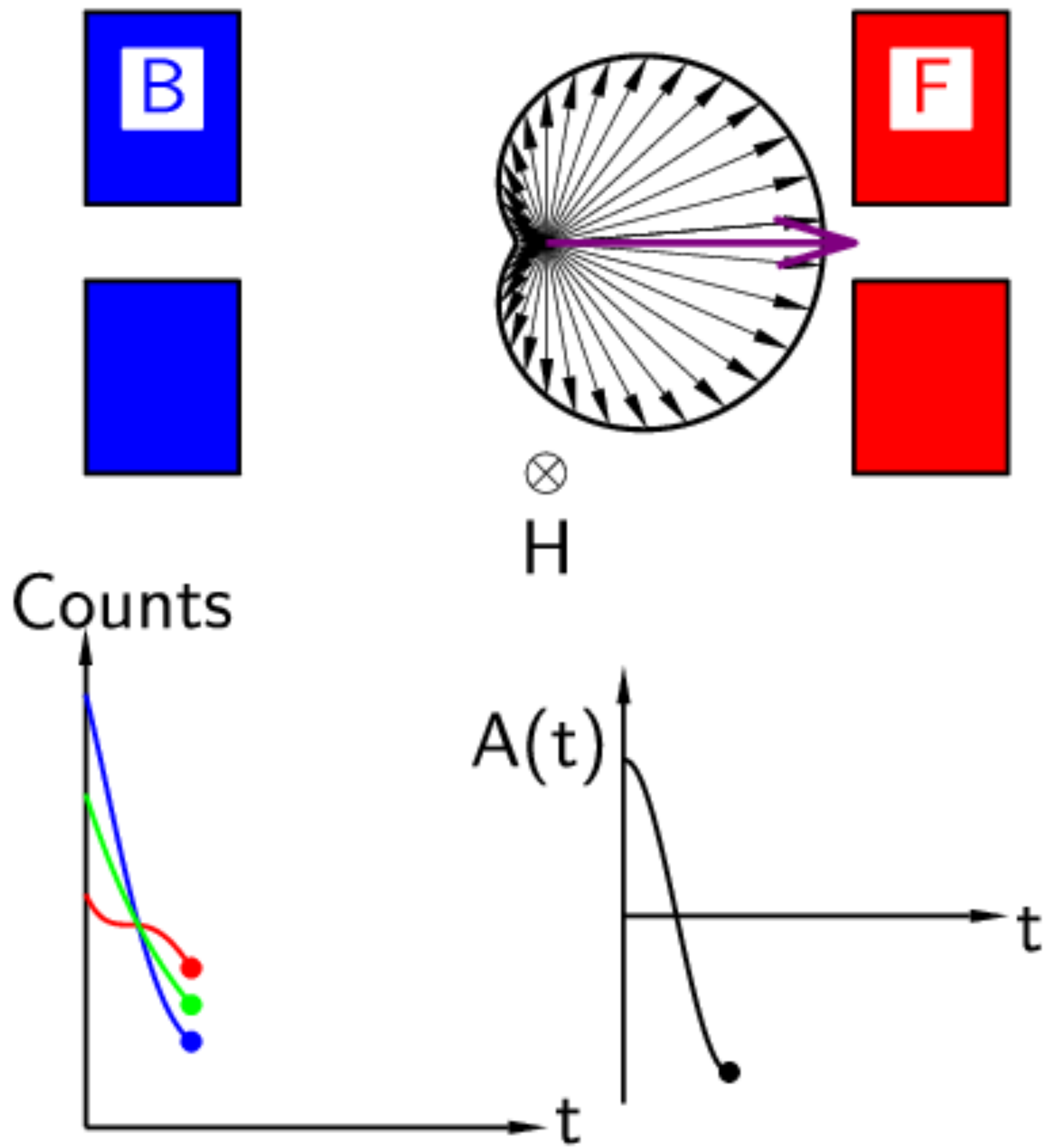
μ^+ decays at rest

$A(t) \propto$ Polarization

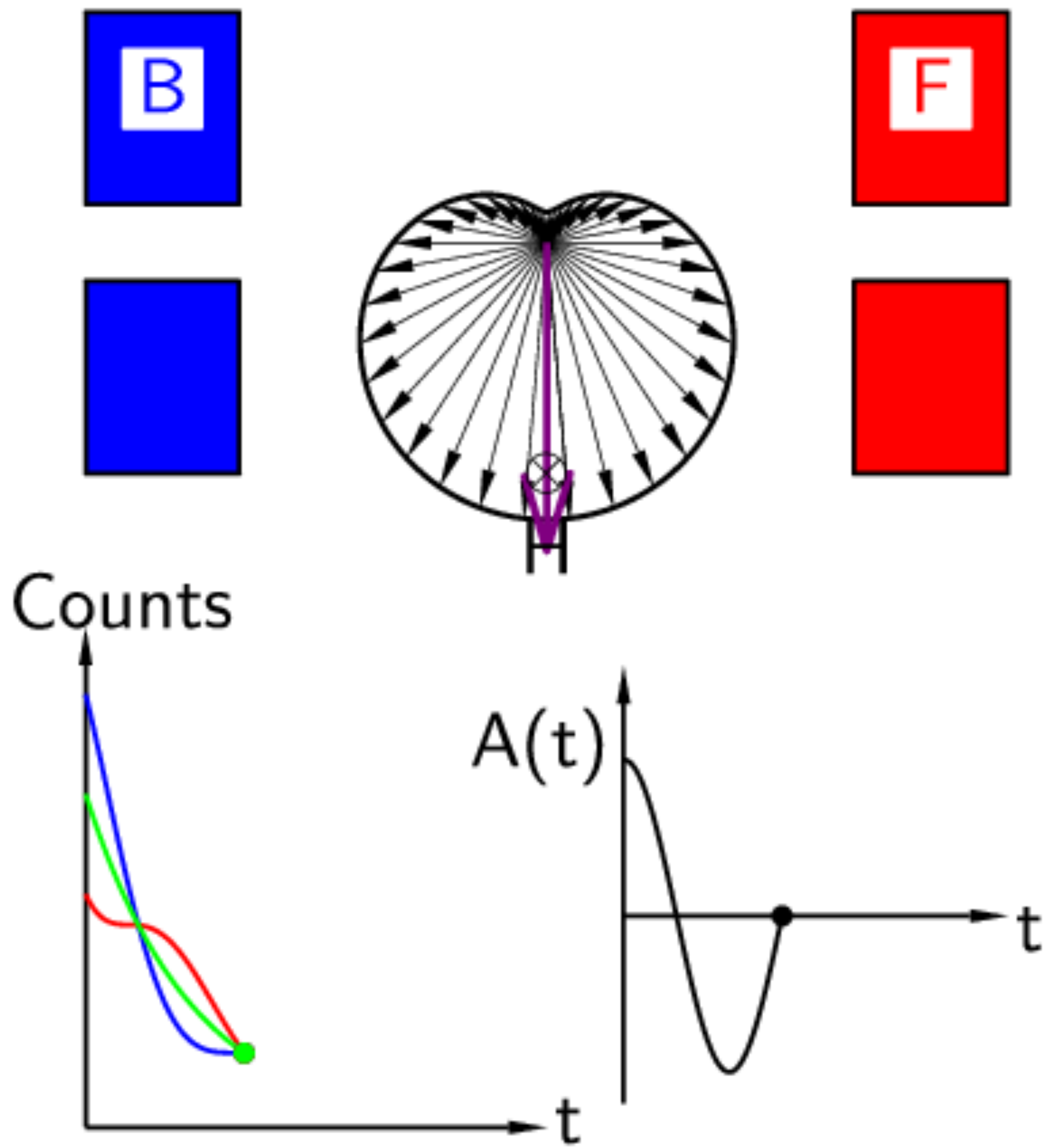

muons
cryostat
quadrupole magnet
Helmholtz magnet
photomultiplier tubes


B
F
H
Counts
t
A(t)
t

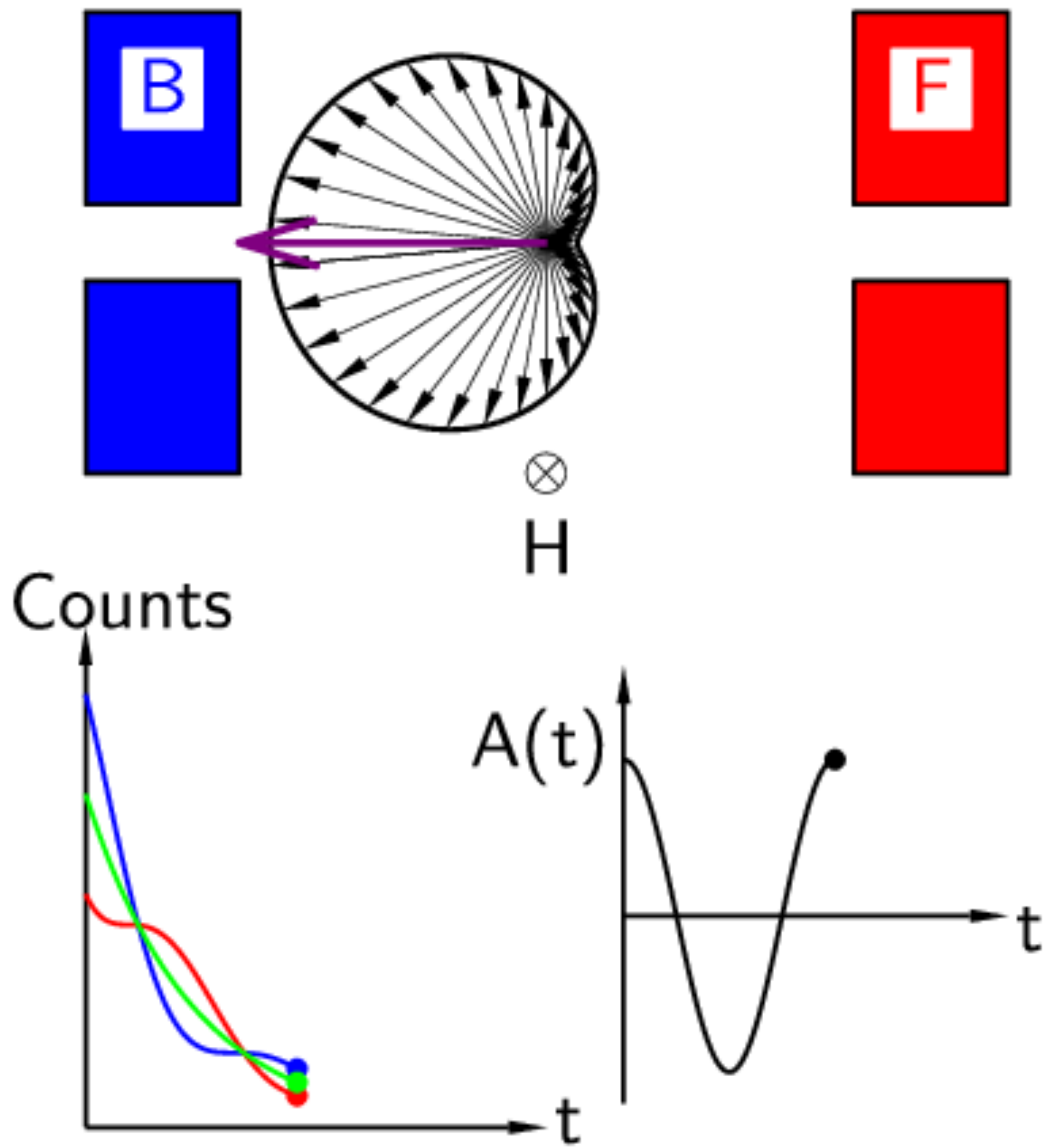

B
F
H
Counts
t
A(t)
t



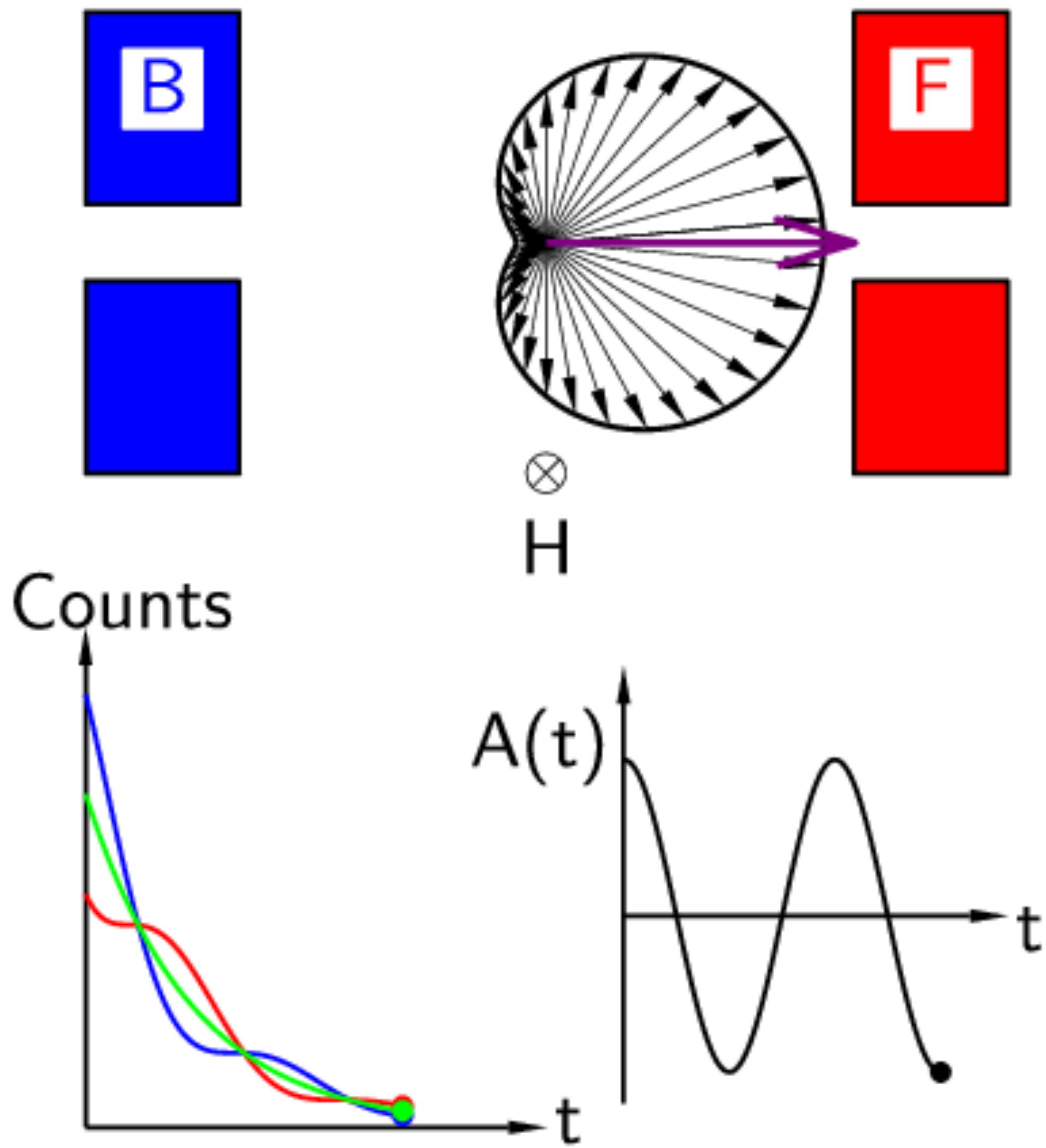
B
F
H
Counts
t
A(t)
t



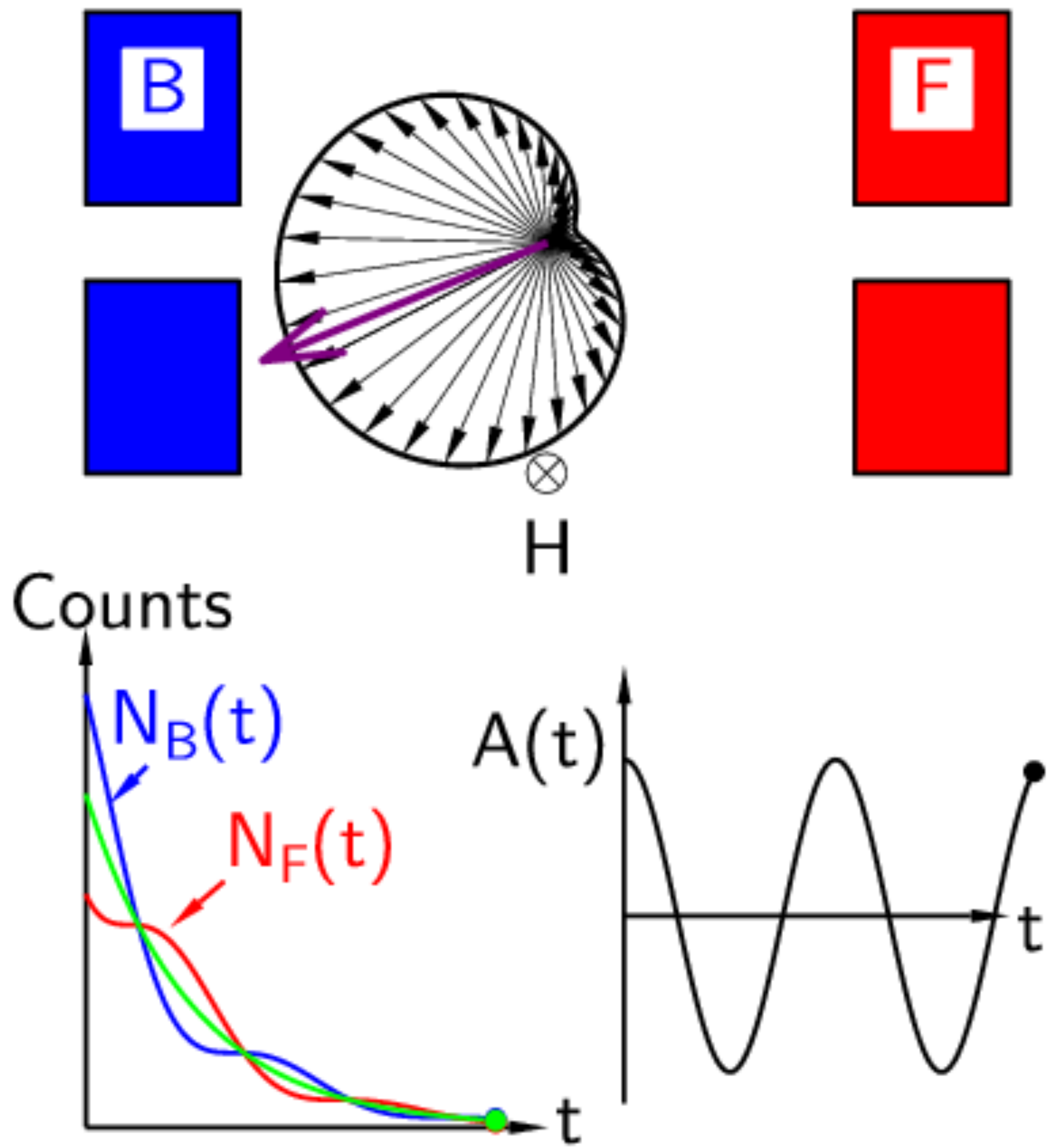
B
F
Counts
t
A(t)
t



B
F
H
Counts
t
A(t)
t



B
F
H
Counts
t
A(t)
t

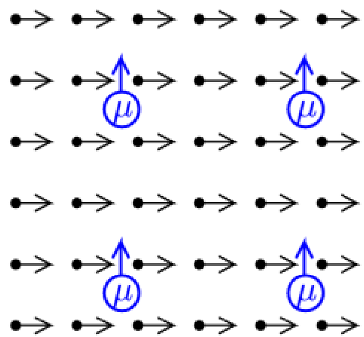


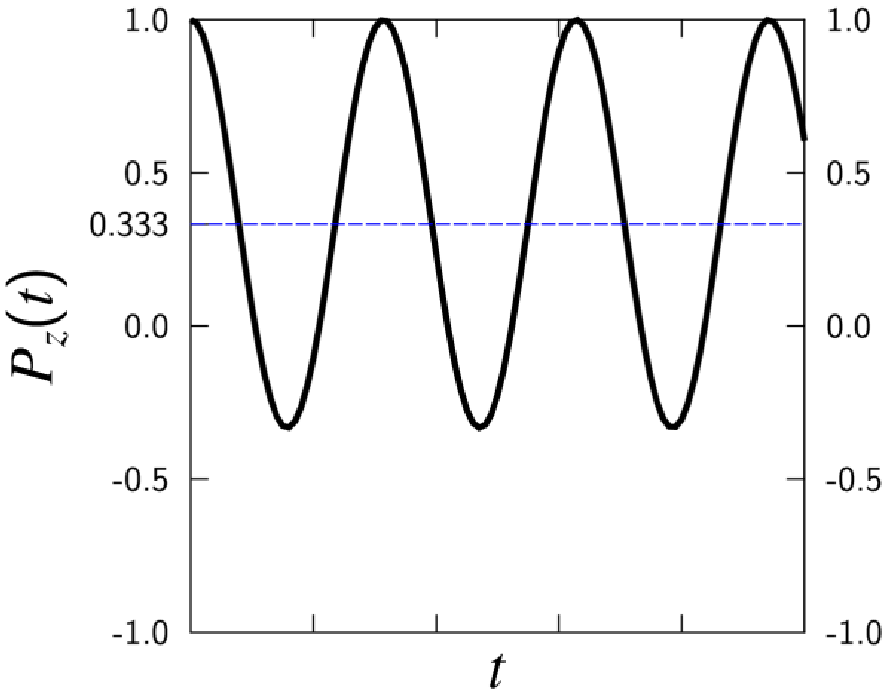
B
F
H
Counts
$N_B(t)$
$N_F(t)$
t
A(t)
t

Typical spectra for polycrystalline samples

Case I:

static order



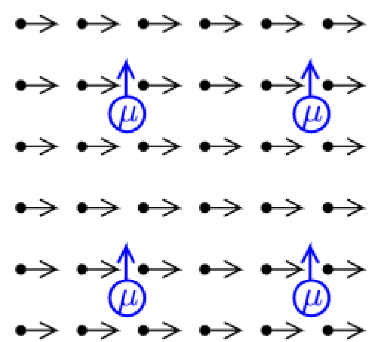


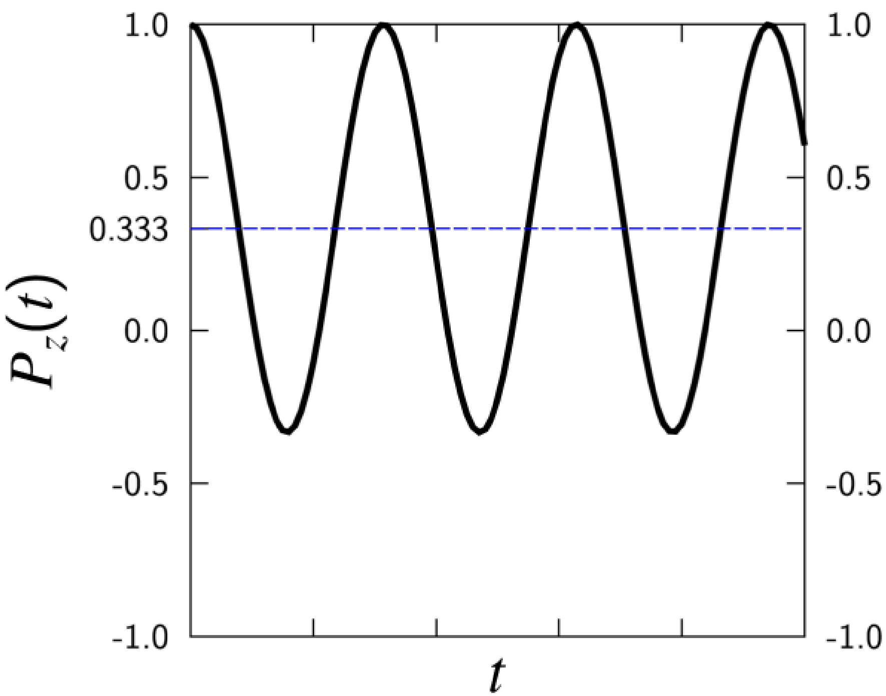
$$P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma B_\mu t)$$

Typical spectra for polycrystalline samples

Case I:

static order

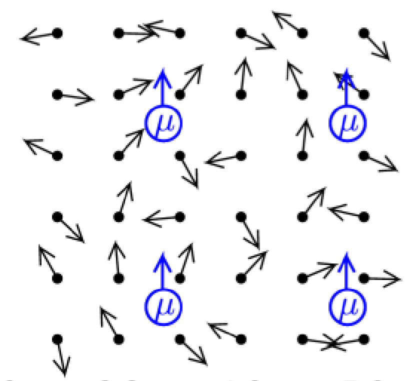


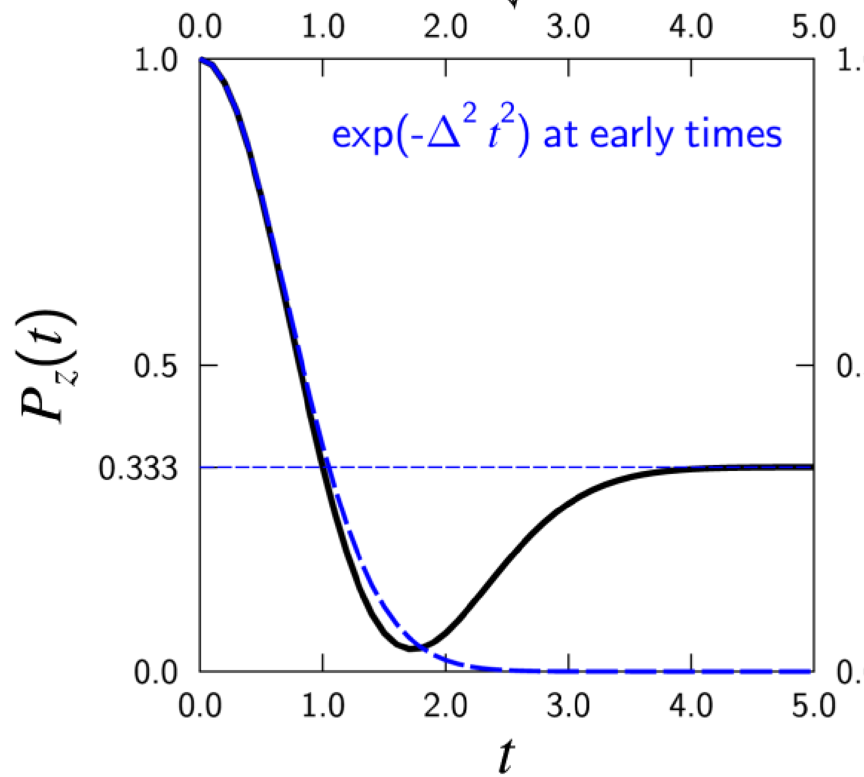


$$P_z(t) = \frac{1}{3} + \frac{2}{3}\cos(\gamma B_\mu t)$$

Case II:

static disorder





$$P_z(t) = \frac{1}{3} + \frac{2}{3}(1 - \Delta^2 t^2)\exp(-\frac{\Delta^2 t^2}{2})$$

More on relaxation functions

$$A(t) \sim \sum_i A_i \cos(\gamma_\mu |B_i| t)$$

muon sites — field at site

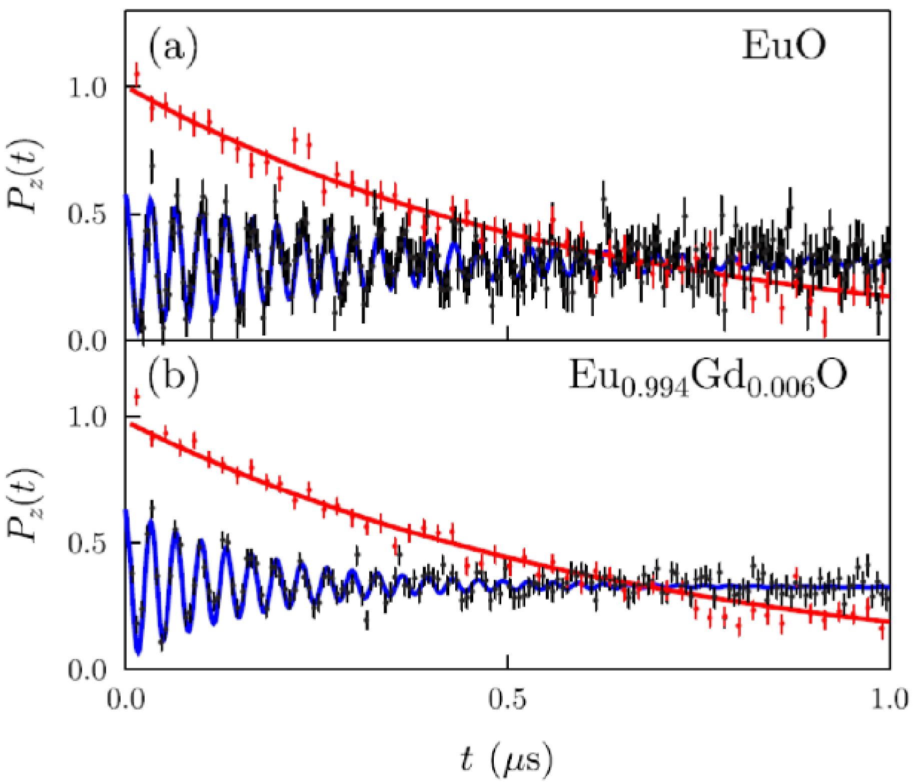
In general $$A(t) \sim \int p(B) \cos(\gamma_\mu B t) \mathrm{d}B$$

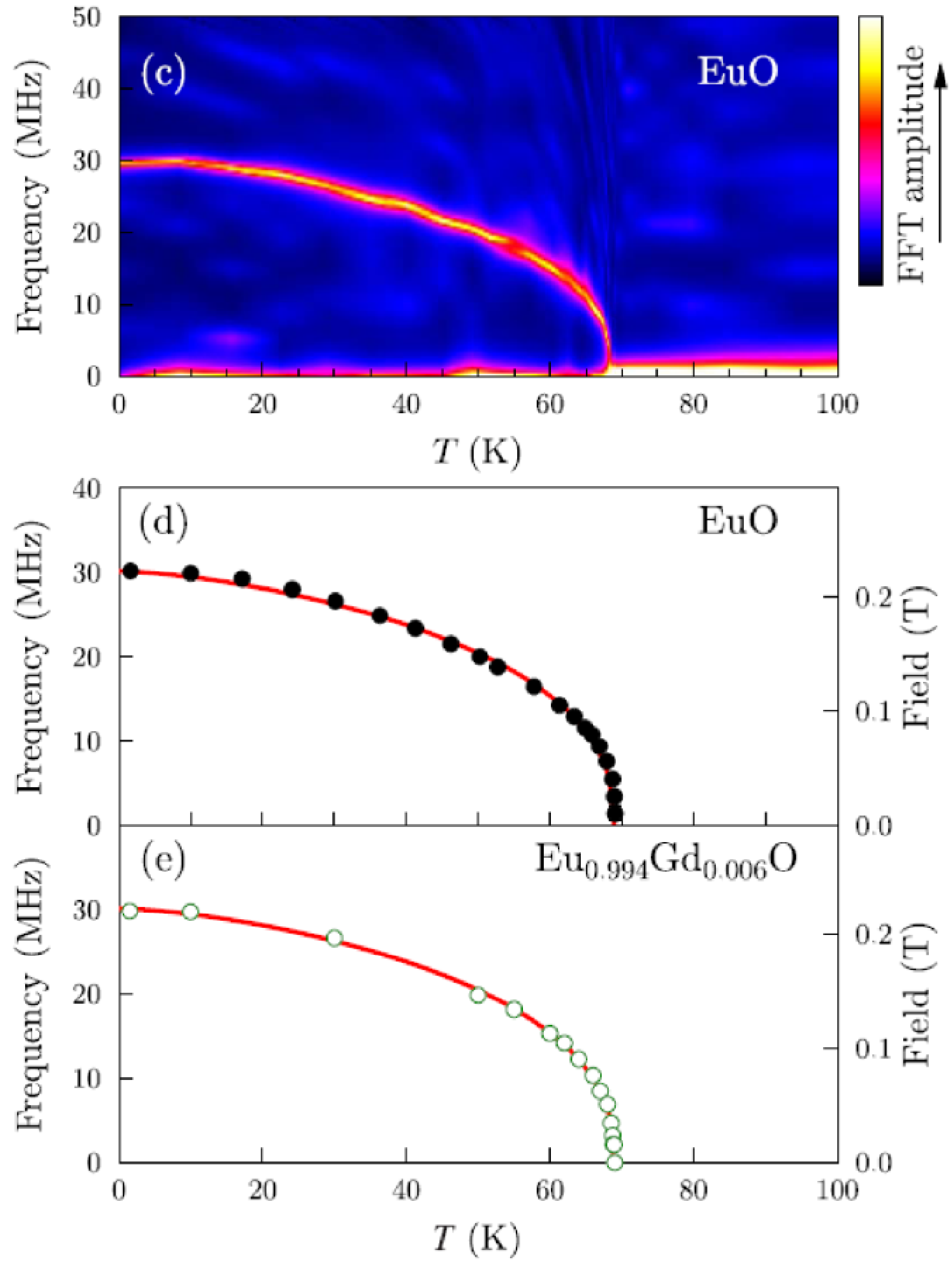
Usually we only have one or two muon sites but we need to take account of broadening/dynamics

$$A(t) = \frac{1}{3}\exp(-\lambda_\parallel t) + \frac{2}{3}\exp(-\lambda_\perp t)\cos(\gamma_\mu B t)$$

$1/T_1$ — $1/T_2$

EuO is THE localized ferromagnet





$$\nu(T) = \nu(0)\left[1 - (T/T_c)^{\alpha}\right]^{\beta}$$

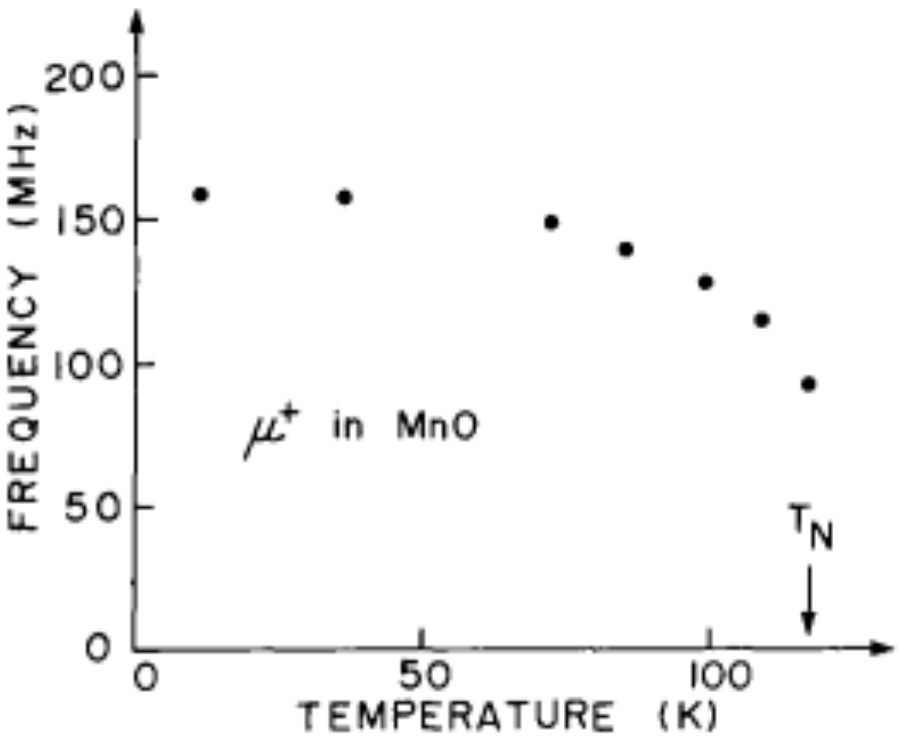
T_c=69.01(1) K

β= 0.32(1)

S.J. Blundell *et al.* PRB **81**, 092407 (2010)

Antiferromagnets

Muons work just as well, since they measure local magnetic fields



Y.J. Uemura *et al.*, Hyperfine Interactions **17**, 339 (1984)

How to fit your precession data

$$\nu = \nu_0 \left[1 - \left(\frac{T}{T_c} \right)^{\alpha} \right]^{\beta}$$

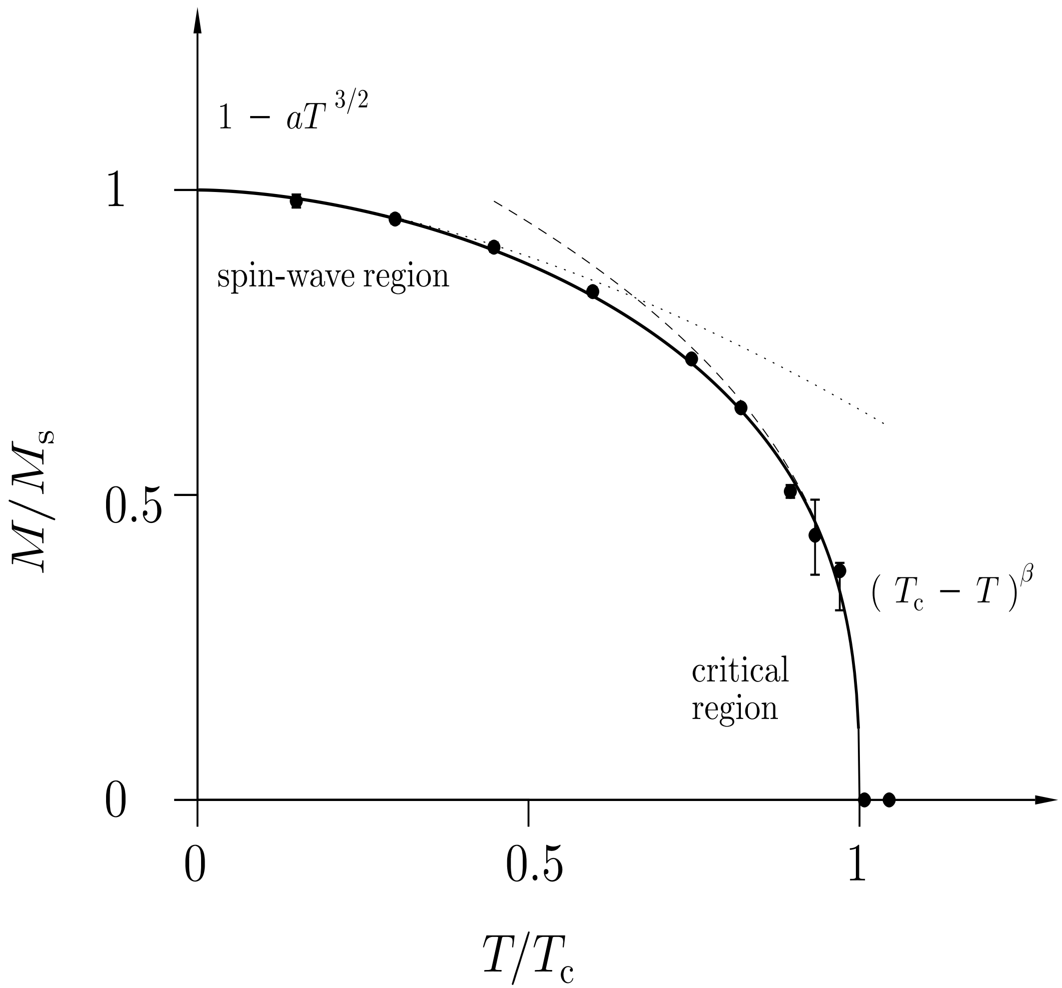
Close to the transition

$$\nu \approx \nu_0 \alpha^{\beta} \left[1 - \frac{T}{T_c} \right]^{\beta}$$

Close to zero temp

$$\nu \approx \nu_0 \left[1 - \beta \left(\frac{T}{T_c} \right)^{\alpha} \right]$$

Excitations of a magnet: Spin waves



How to fit your precession data

$$\nu = \nu_0 \left[1 - \left(\frac{T}{T_\mathrm{c}} \right)^\alpha \right]^\beta$$

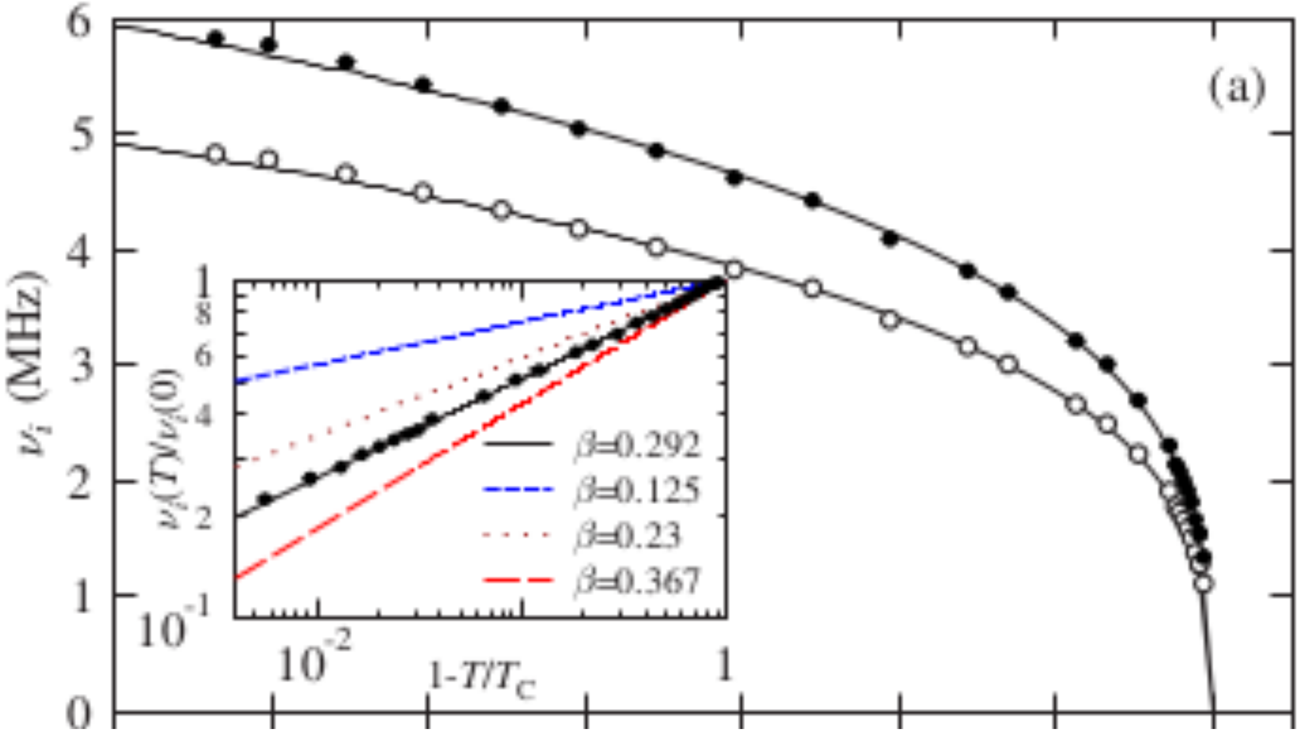
Close to the transition

$$\nu \approx \nu_0 \alpha^\beta \left[1 - \frac{T}{T_\mathrm{c}} \right]^\beta$$

Close to zero

$$\nu \approx \nu_0 \left[1 - \beta \left(\frac{T}{T_\mathrm{c}} \right)^\alpha \right]$$

Scaling plots

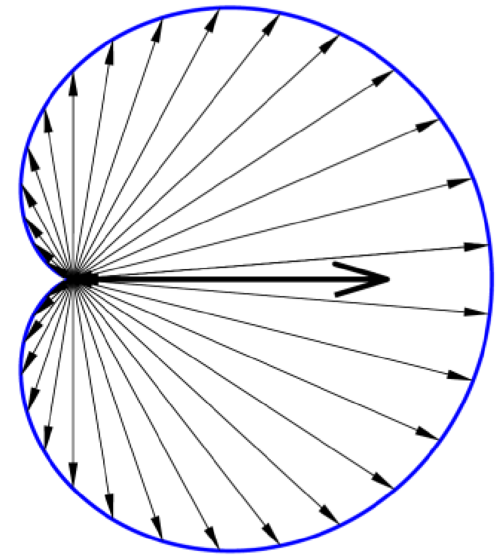


Example: Cs_2AgF_4

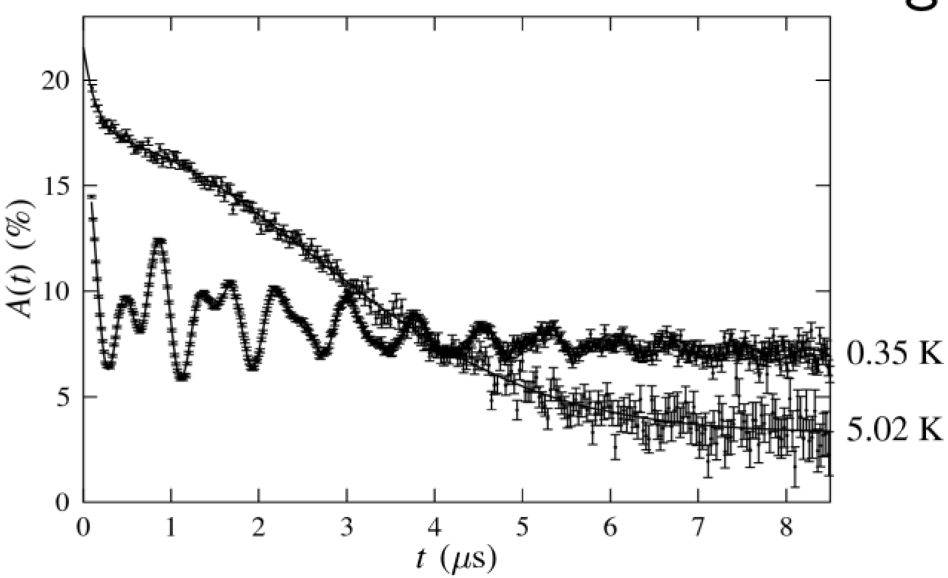
Phys Rev B **75,** 220408 (2007)

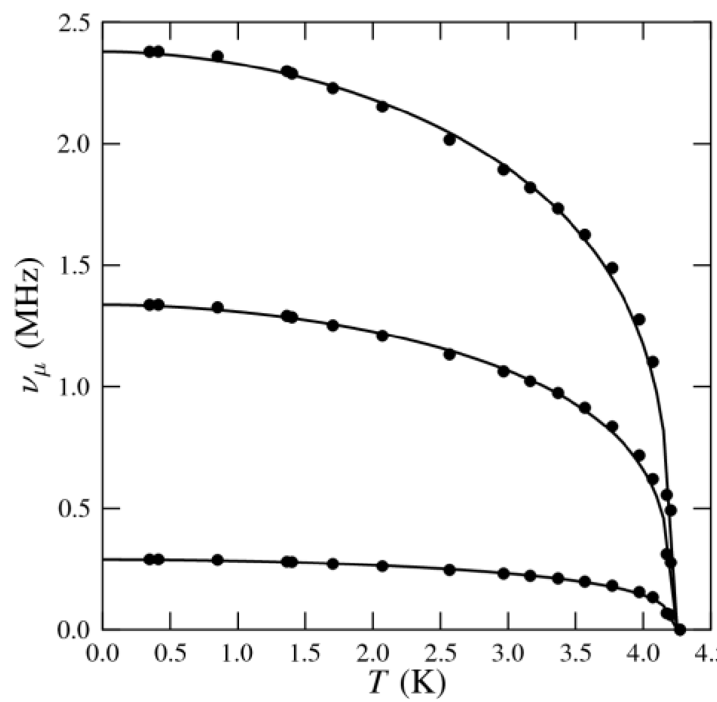
Muons as a probe of magnetism

- Microscopic: sensitive to local effects
- Sensitive to very weak magnetism
- Work well in zero applied field
- One muon at a time $\longrightarrow$ ultra dilute!

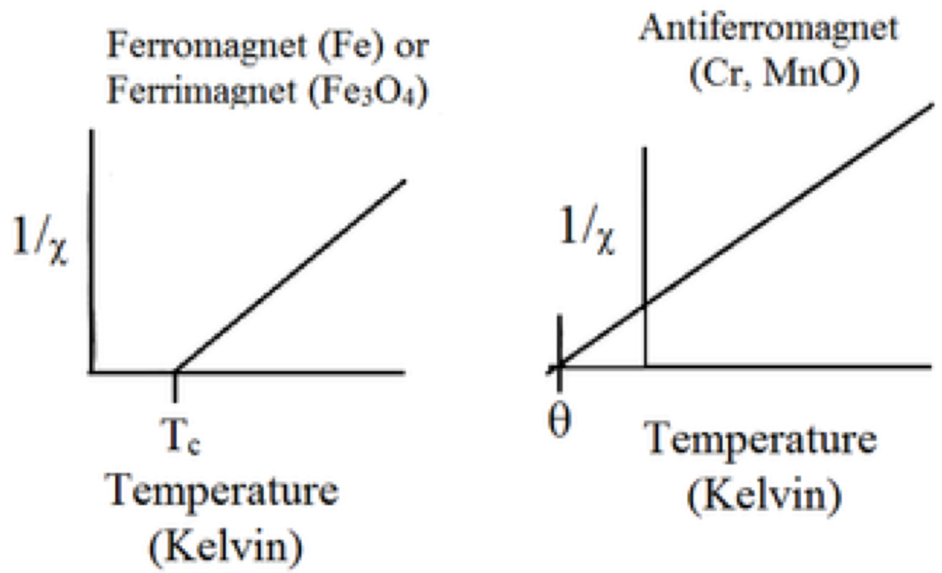


- μ^+SR is great for: small moment magnetism
  random magnetism
  short range effects



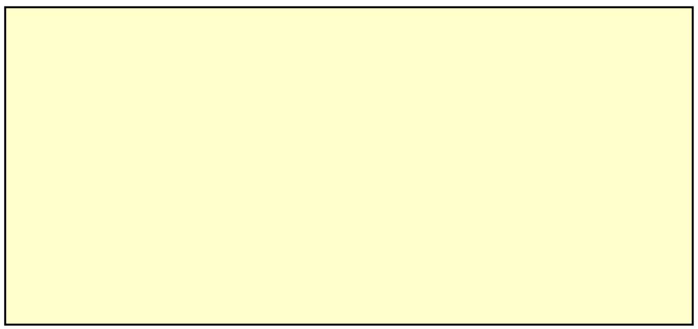


Usually we make susceptibility measurements before we make measurements with muons

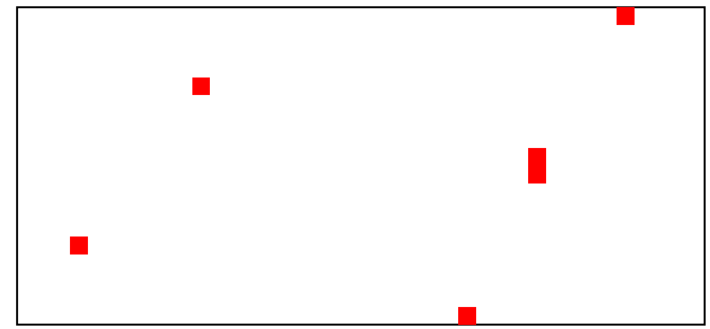


Uniformly weakly magnetic

Non-magnetic, with strongly magnetic impurities



or



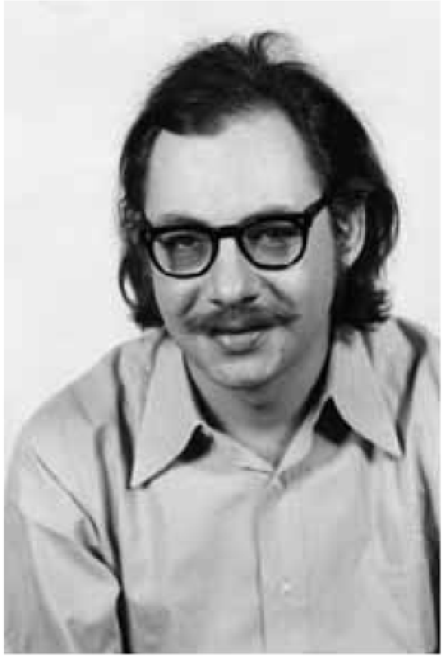
<u>Susceptibility</u> gives average information and therefore can give the same response for the situations sketched above

<u>*μ*SR</u> gives local information and therefore can distinguish between these two situations.

Case study

One dimensional molecular magnets

Models of low dimensional magnetism



	$D=1$, Ising	$D=2$, XY	$D=3$, Heisenberg
$d=1$	no order	no order	no order
$d=2$	order	no order	no order
$d=3$	order	order	order

$D=$ Dimension of the spins

$d=$dimension of the lattice

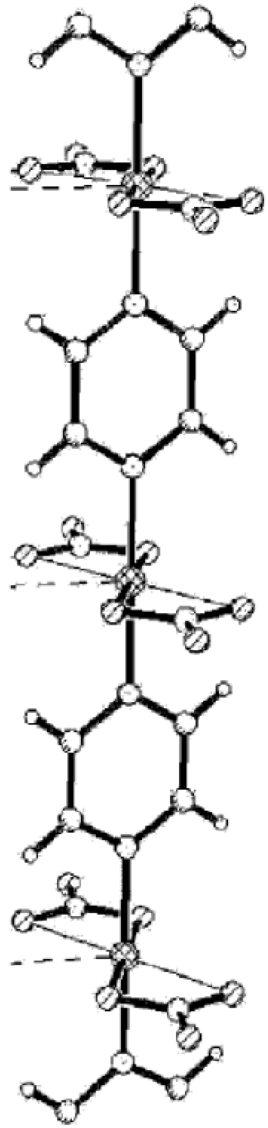
Coleman-Mermin-Wagner theorem forbids breaking a continuous symmetry for $d=1$ and 2 for $T>0$

We can describe the physics with a deceptively simple looking equation

$$H = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

Anderson *Basic notions in Condensed Matter Physics*

$Cu(NO_3)_2(pyz)$ Magnetism in 1 dimension

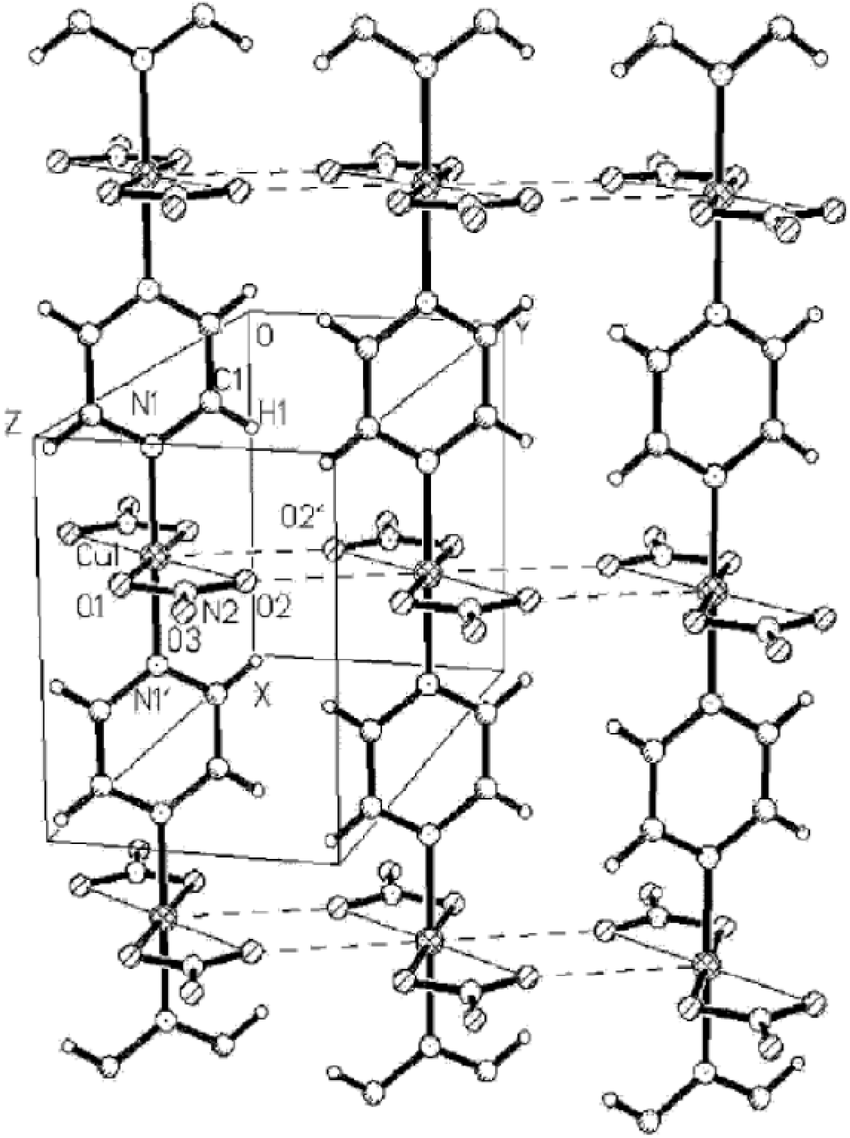


S=1/2 Cu^{2+} ions linked by pyz

1D Cu-(pyz)-Cu chains along a

A Santoro *et al.*, Acta. Cryst., **95** 5780 (1973)
P R Hammar *et al.*, Phys. Rev. B, **59** 1008 (1999)

$Cu(NO_3)_2(pyz)$ Magnetism in 1 dimension

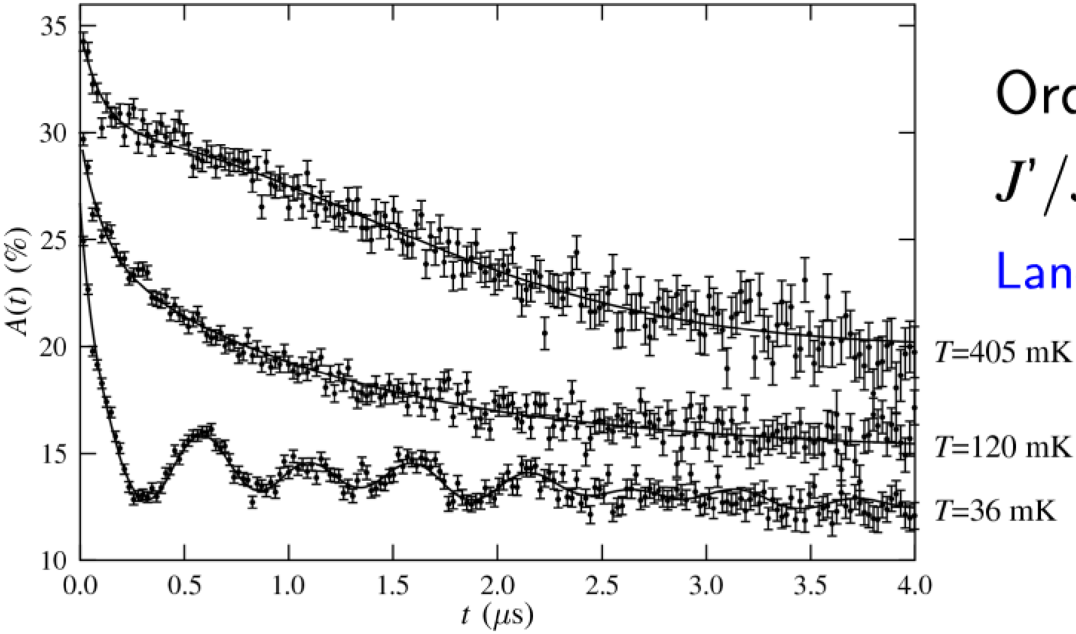


- $S = 1/2$ Cu^{2+} ions linked by pyz
- 1D Cu-(pyz)-Cu chains along a
- High field magnetization and specific heat give $|J|/k_B = 10.3$ K
- No evidence of magnetic order down to 70 mK

A Santoro *et al.*, Acta. Cryst., **95** 5780 (1973)
P R Hammar *et al.*, Phys. Rev. B, **59** 1008 (1999)

Molecular magnets: muons are unique!

Observation of magnetic order - invisible to other techniques

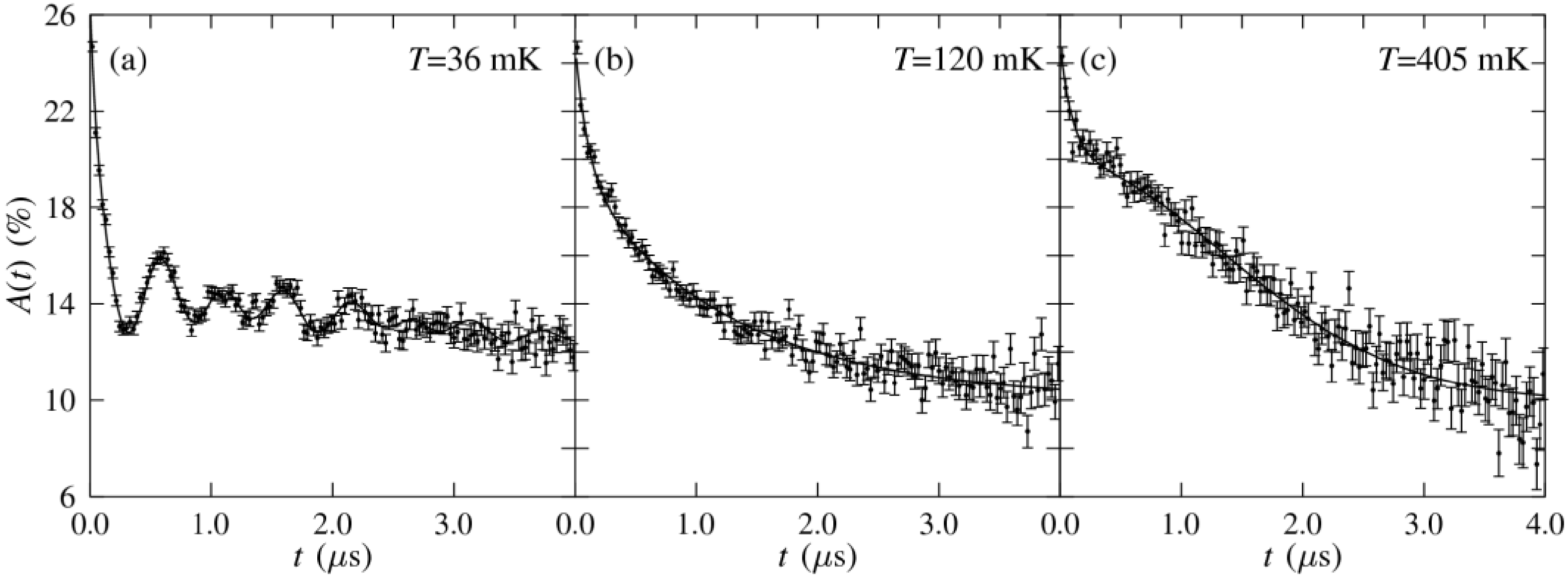


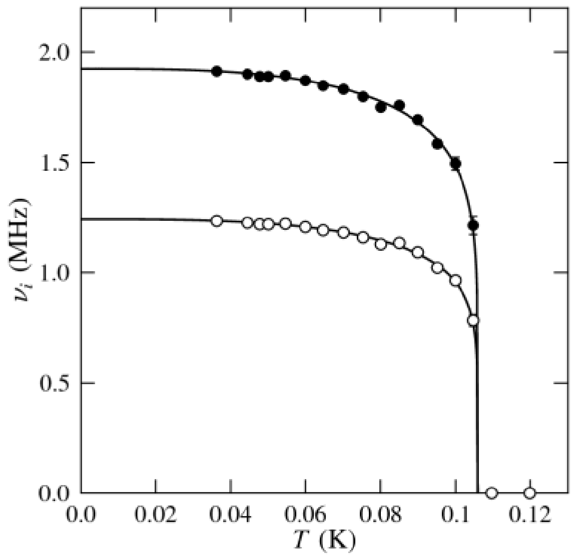
Order observed in CuPzN with T_{N}=107 mK

J'/J=4.4 $\times 10^{-3}$

Lancaster *et al.* Phys. Rev. B, **73** 020410(R) (2006)

$Cu(NO_3)_2(pyz)$ μ^+SR results



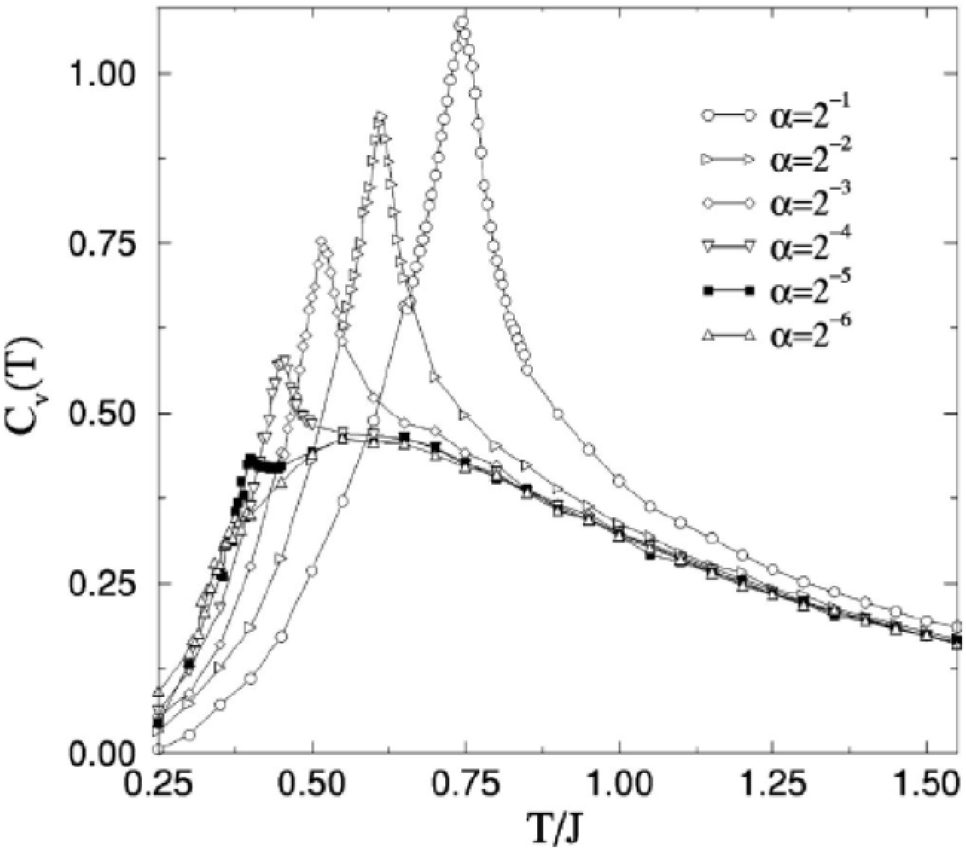


We detect magnetic order with μ^+SR

T_N=107 mK

T Lancaster *et al.* Phys. Rev. B, **73** 020410 (2006)

The problem with finding T_N in low-d systems



Stochastic series QMC simulations say

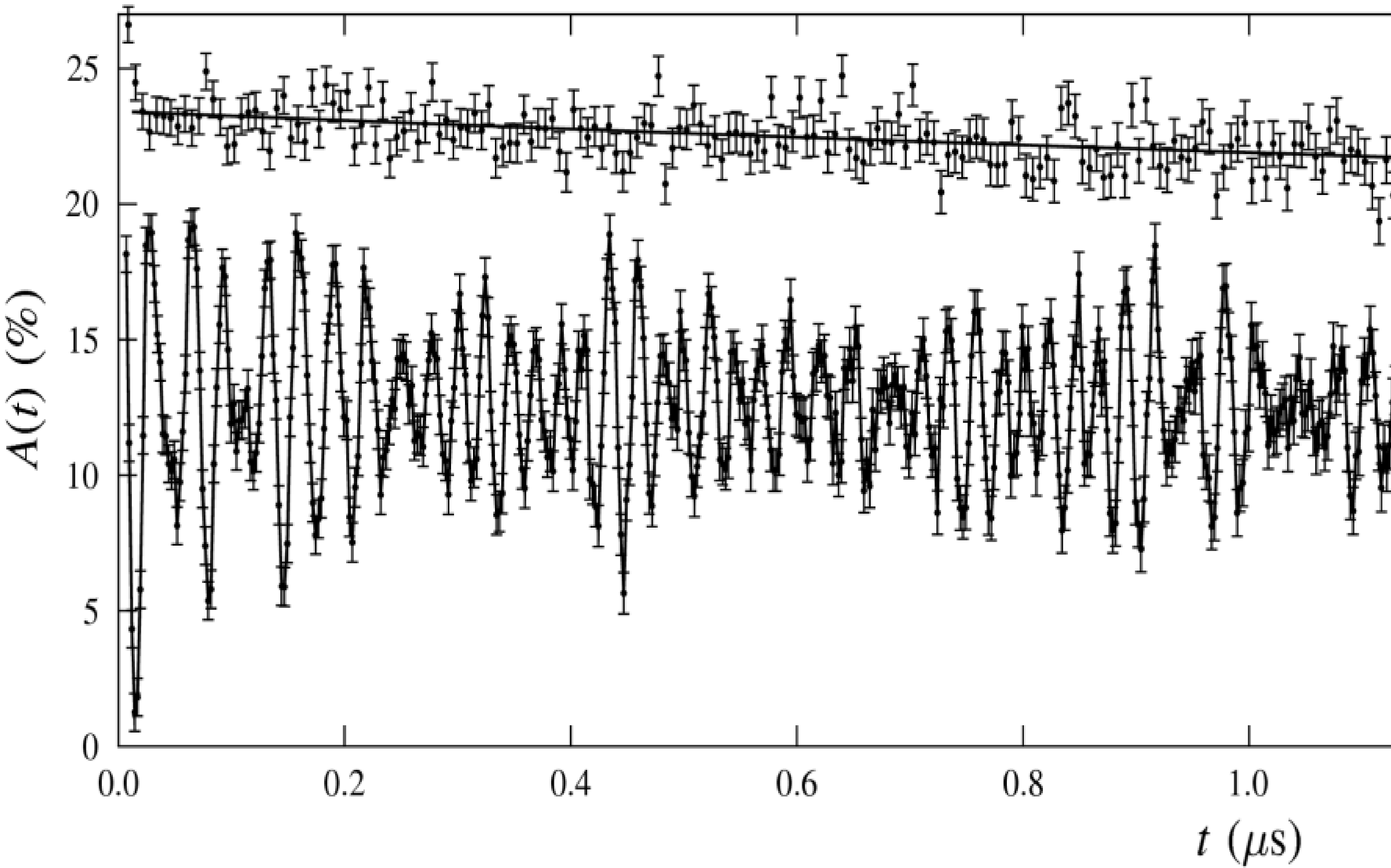
The anomaly in C_v decreases with decreasing α

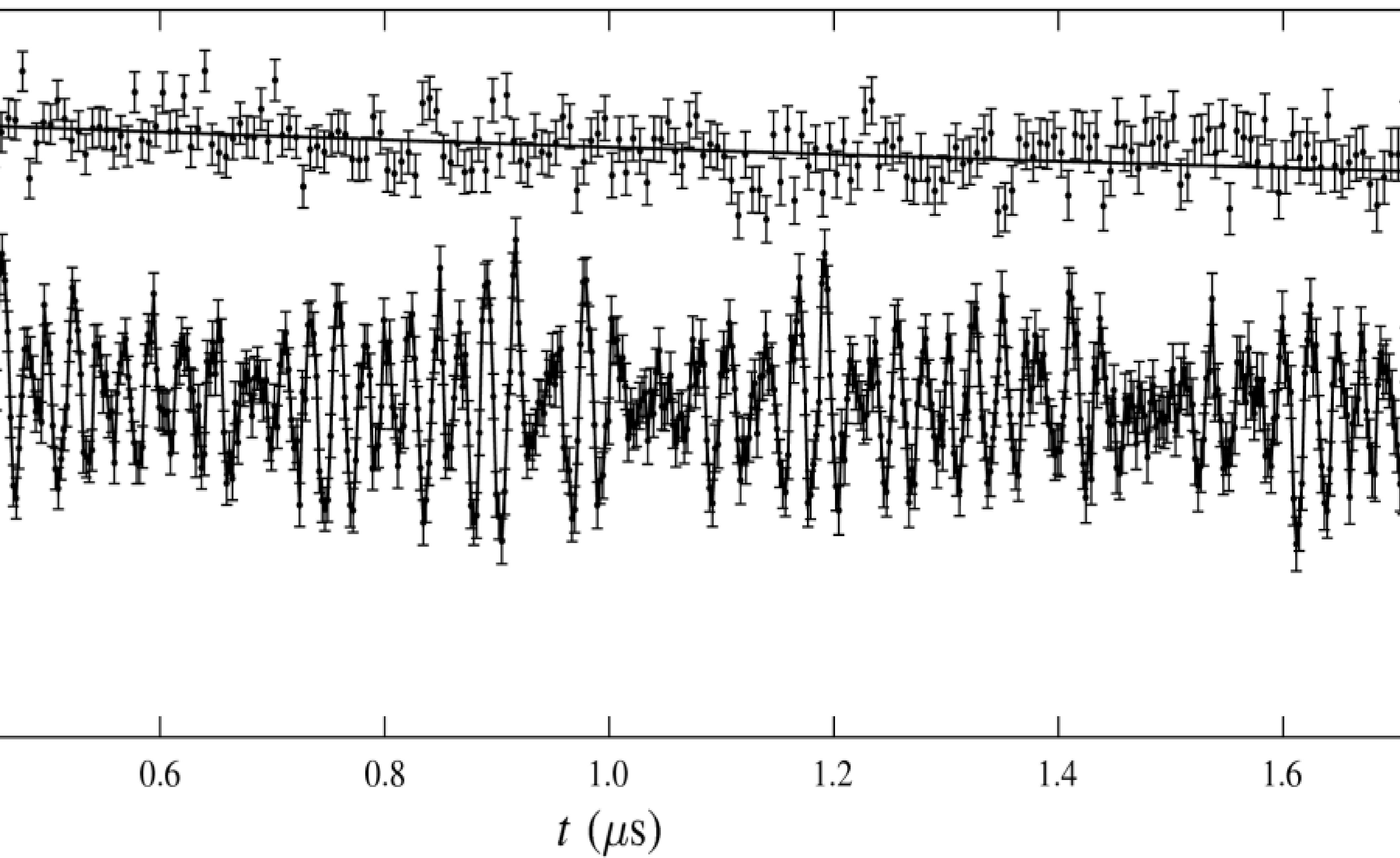
This is due to correlations above T_N (ΔS at T_N is therefore reduced)

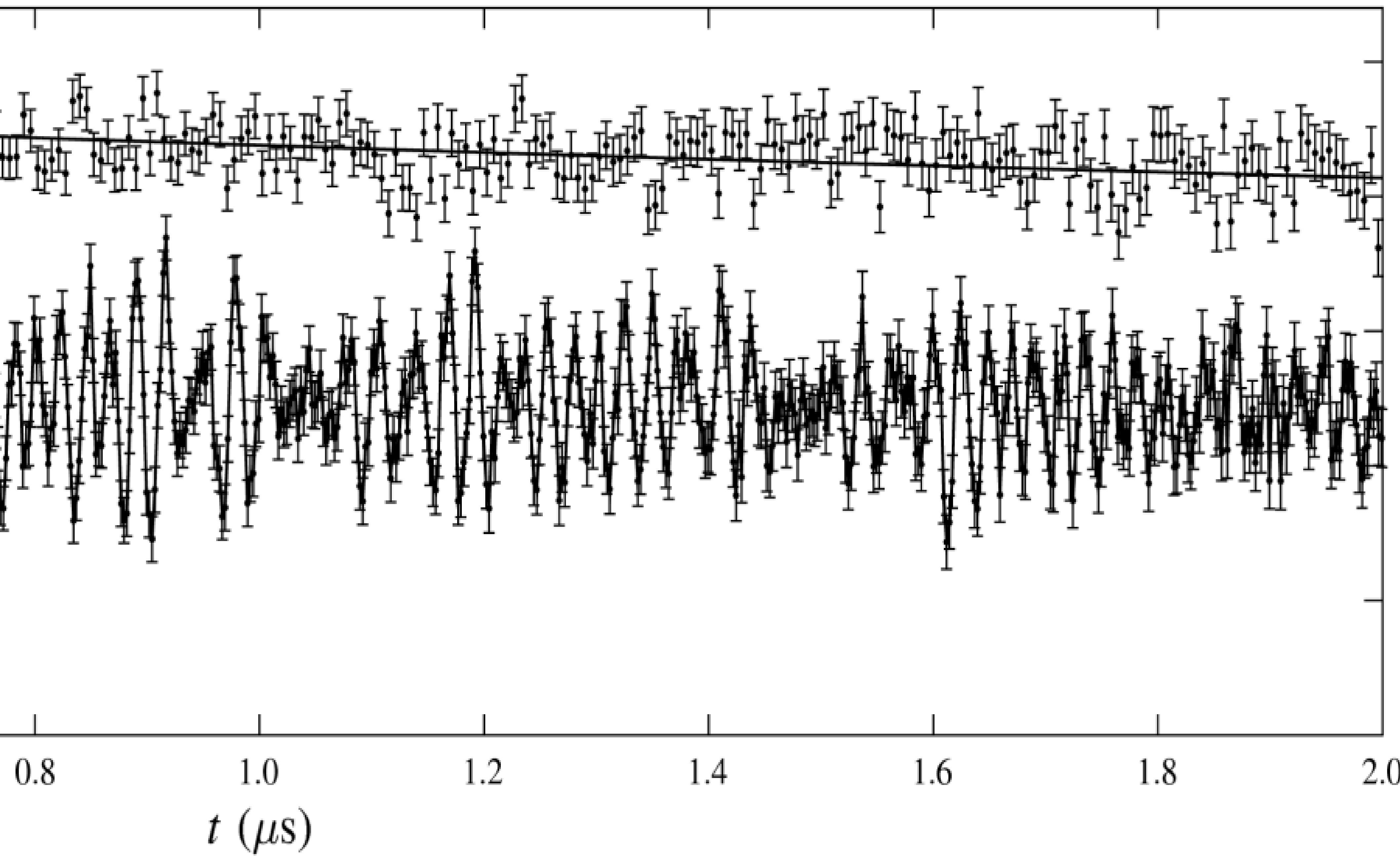
Other measurements made difficult by the small magnetic moment in anisotropic systems

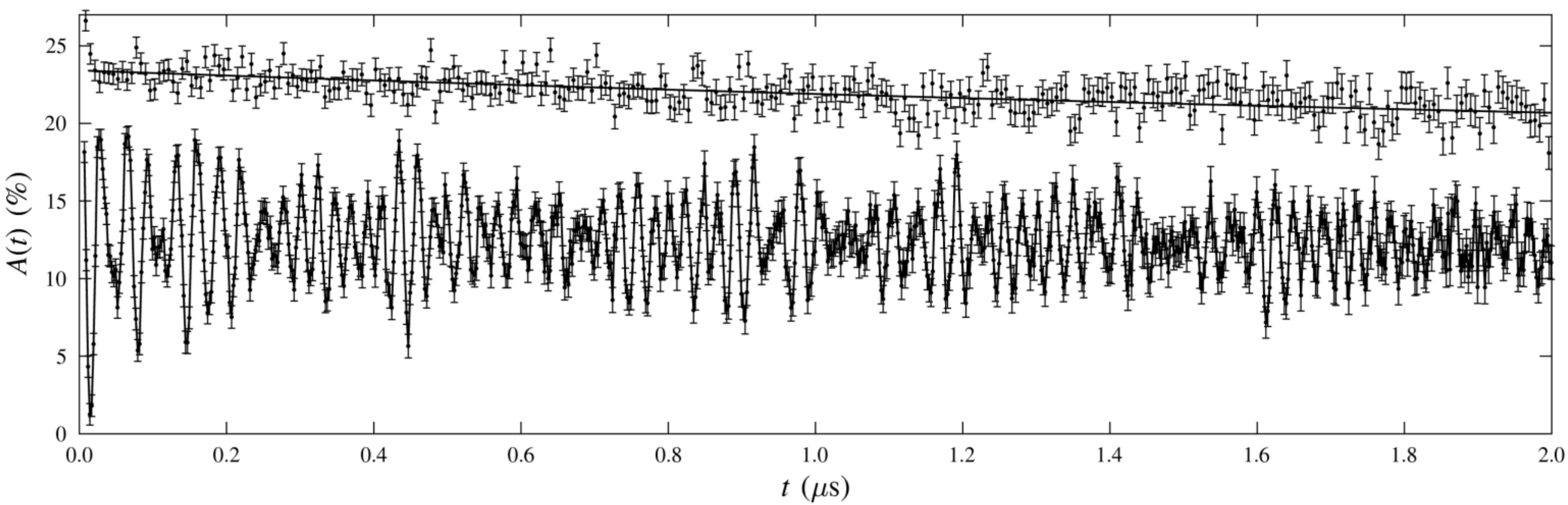
P Sengupta *et al.* Phys Rev B **68** 94423 (2003)

The most beautiful magnetic spectrum ever measured?

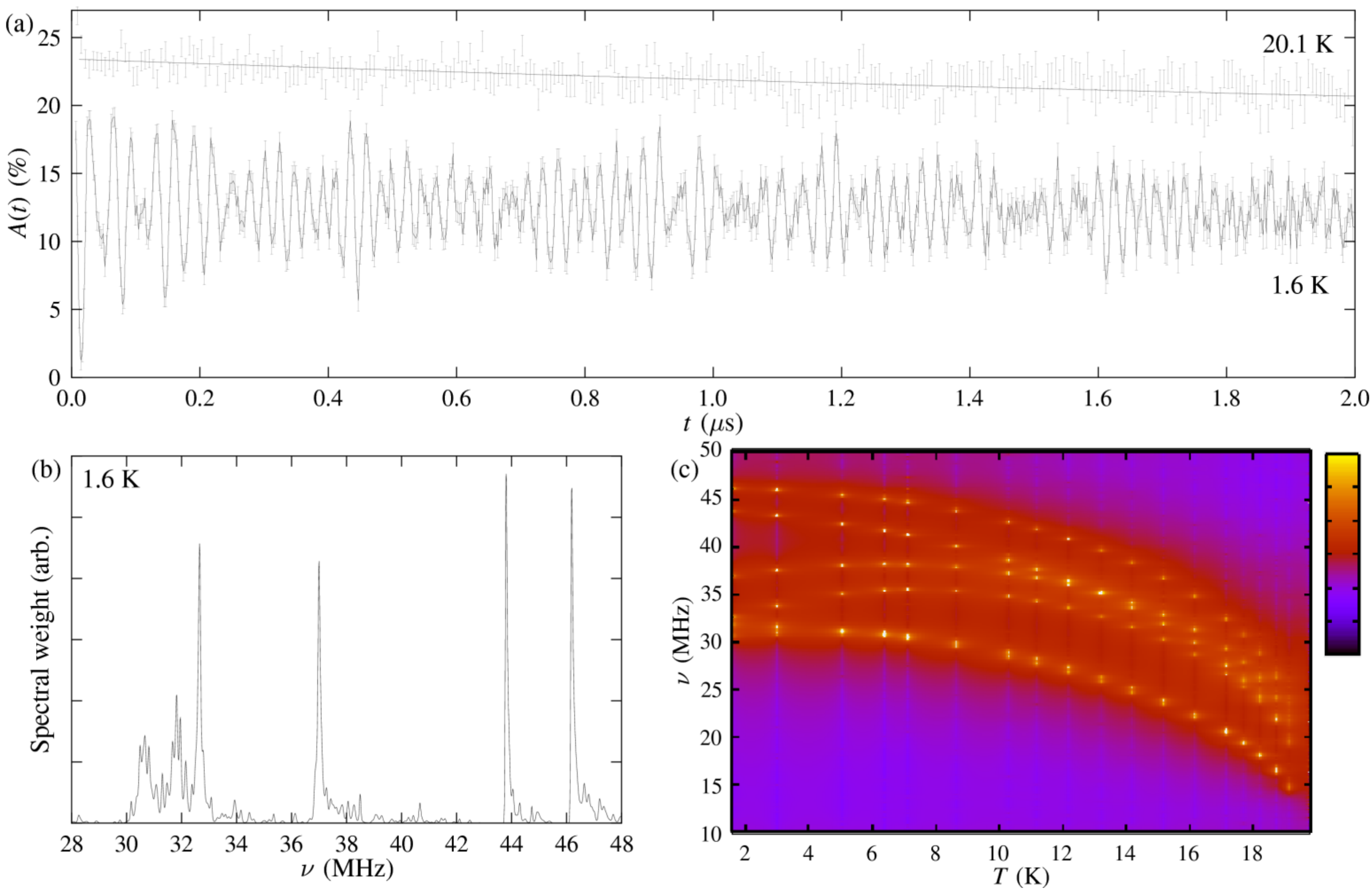



0.6
0.8
1.0
1.2
1.4
1.6
t (μs)

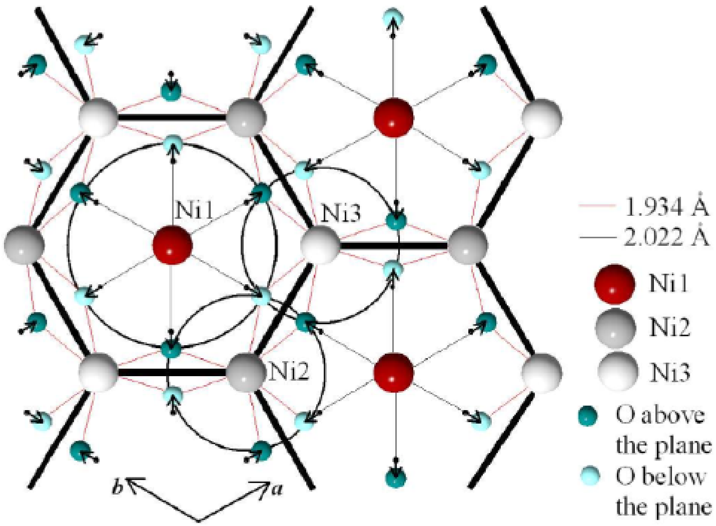

0.8
1.0
1.2
1.4
1.6
1.8
2.0
t (μs)

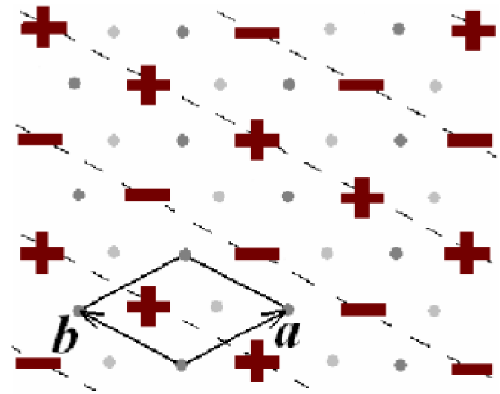


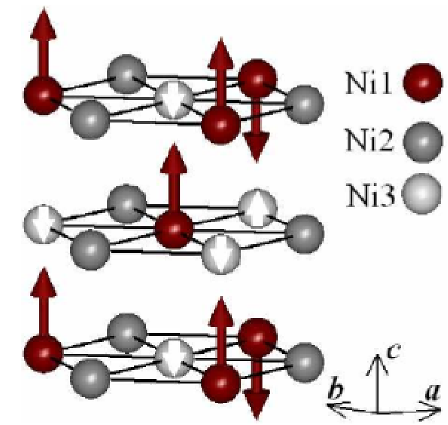
25
20
15
10
5
0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
A(t) (%)
t (μs)

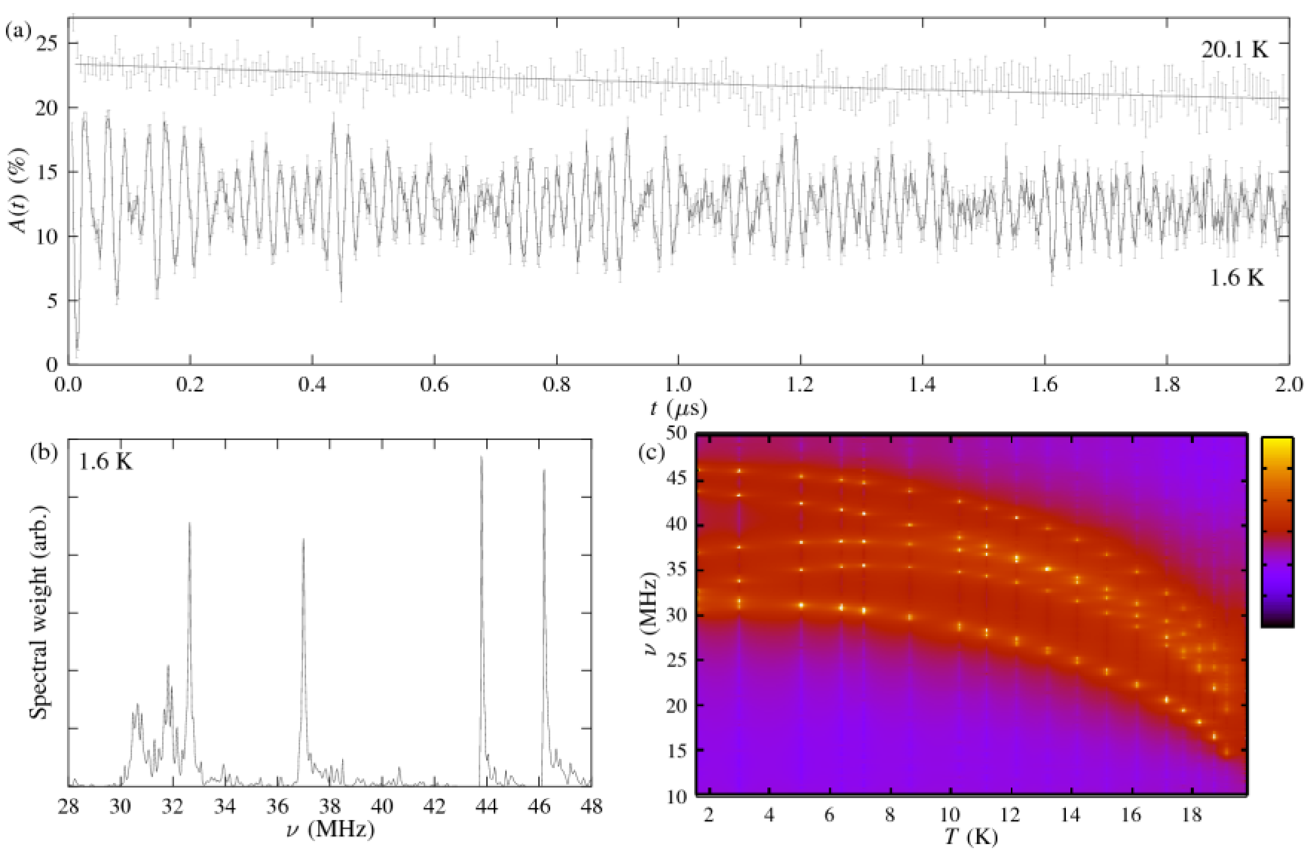

(a)
20.1 K
1.6 K
A(t) (%)
t (μs)
(b)
1.6 K
Spectral weight (arb.)
ν (MHz)
(c)
ν (MHz)
T (K)

AgNiO$_2$: a new charge ordered state of matter?









Orbital degeneracy lifted via a charge ordering mechanism

This gives rise to a well defined magnetic structure

Muons see this, but show an anomalous T dependence

Lancaster *et al.*, PRL **100** 017206 (2008)

What on earth are we measuring?

Local magnetic field at the muon site

* $B_L = \frac{\mu_0 M}{3}$ LORENTZ FIELD

  site independent
  zero for antiferromagnets

* $B_{dip}(\underline{r}_\mu)$ DIPOLAR FIELD

  depends on muon site
  depends on direction of $\underline{M}$

* $B_{hf}(\underline{r}_\mu)$ HYPERFINE FIELD

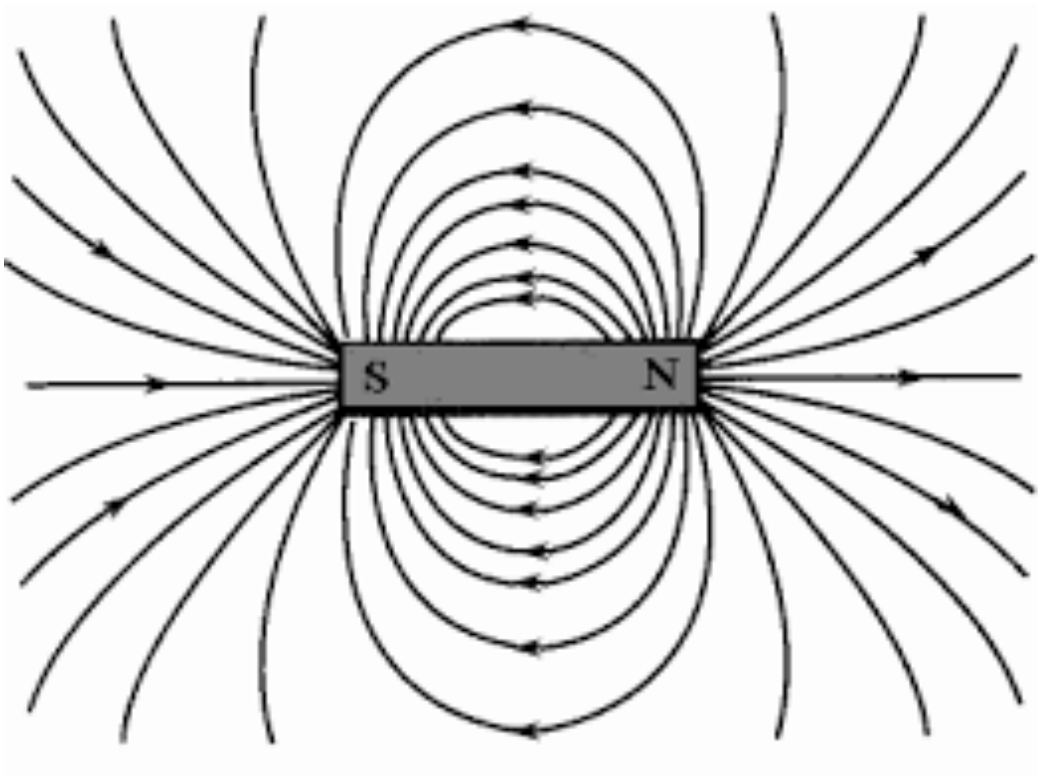
  due to electron spin density at muon site

* B_{demag} DEMAGNETIZATION FIELD

  depends on sample shape or domain structure

Dipole field

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\boldsymbol{m}}{r^3}\right].$$



Dipole field

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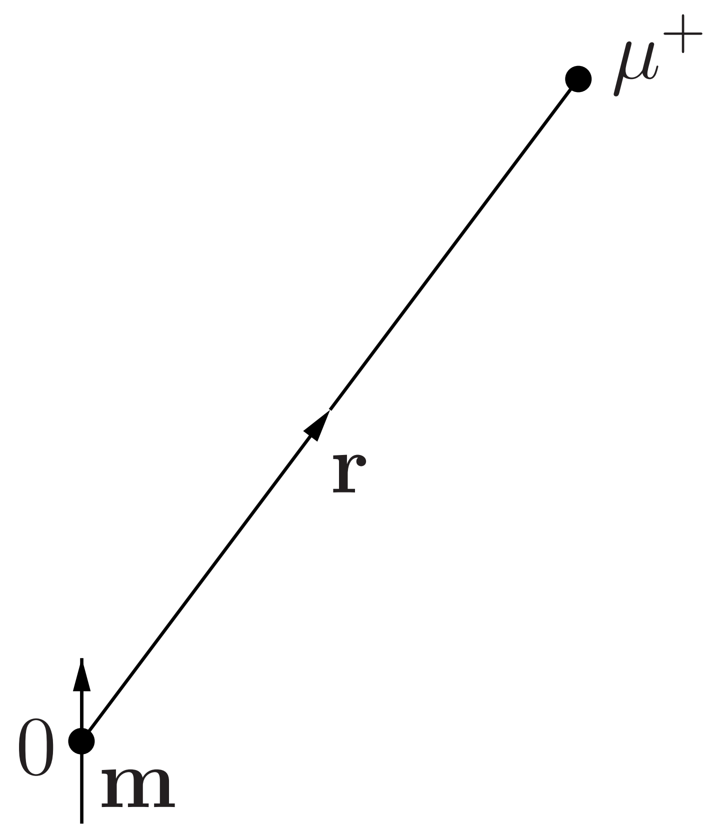


Dipole field

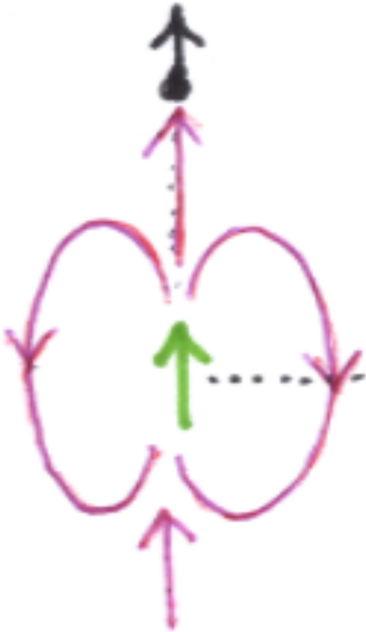
$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\boldsymbol{m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\boldsymbol{m}}{r^3}\right].$$

$$\boldsymbol{B}(\boldsymbol{r}) = \hat{\boldsymbol{D}}(\boldsymbol{r})\boldsymbol{m},$$

$$\hat{\boldsymbol{D}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\begin{pmatrix} -\frac{1}{r^3}+\frac{3r_x^2}{r^5} & \frac{3r_xr_y}{r^5} & \frac{3r_xr_z}{r^5} \\ \frac{3r_yr_x}{r^5} & -\frac{1}{r^3}+\frac{3r_y^2}{r^5} & \frac{3r_yr_z}{r^5} \\ \frac{3r_zr_x}{r^5} & \frac{3r_zr_y}{r^5} & -\frac{1}{r^3}+\frac{3r_z^2}{r^5} \end{pmatrix}.$$

Dipole field: example 1: muon separated from a moment along *z*

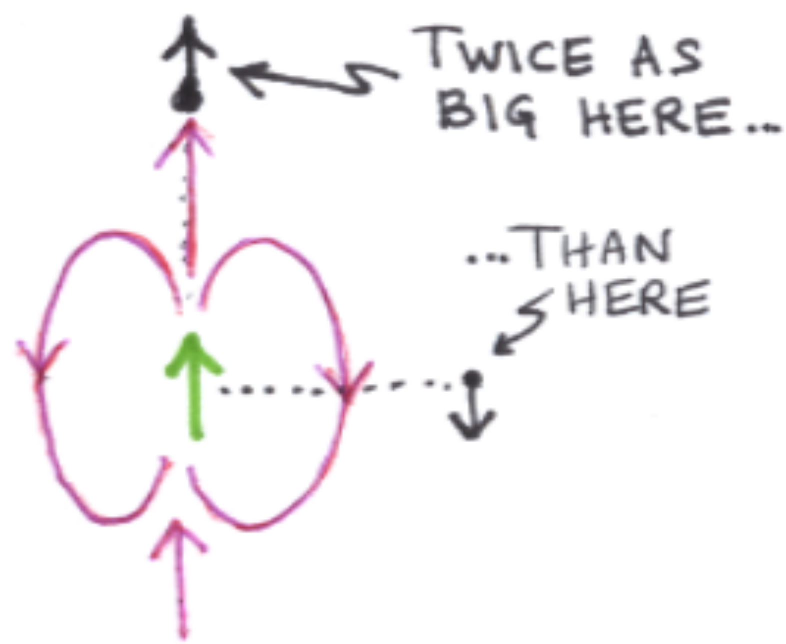
$$\boldsymbol{D} = \frac{\mu_0}{4\pi} \begin{pmatrix} -\frac{1}{z^3} & 0 & 0 \\ 0 & -\frac{1}{z^3} & 0 \\ 0 & 0 & \frac{2}{z^3} \end{pmatrix}$$



$$\boldsymbol{B}(0,0,z) = \frac{\mu_0}{4\pi} \left(-\frac{m_x}{z^3}, -\frac{m_y}{z^3}, \frac{2m_z}{z^3} \right).$$

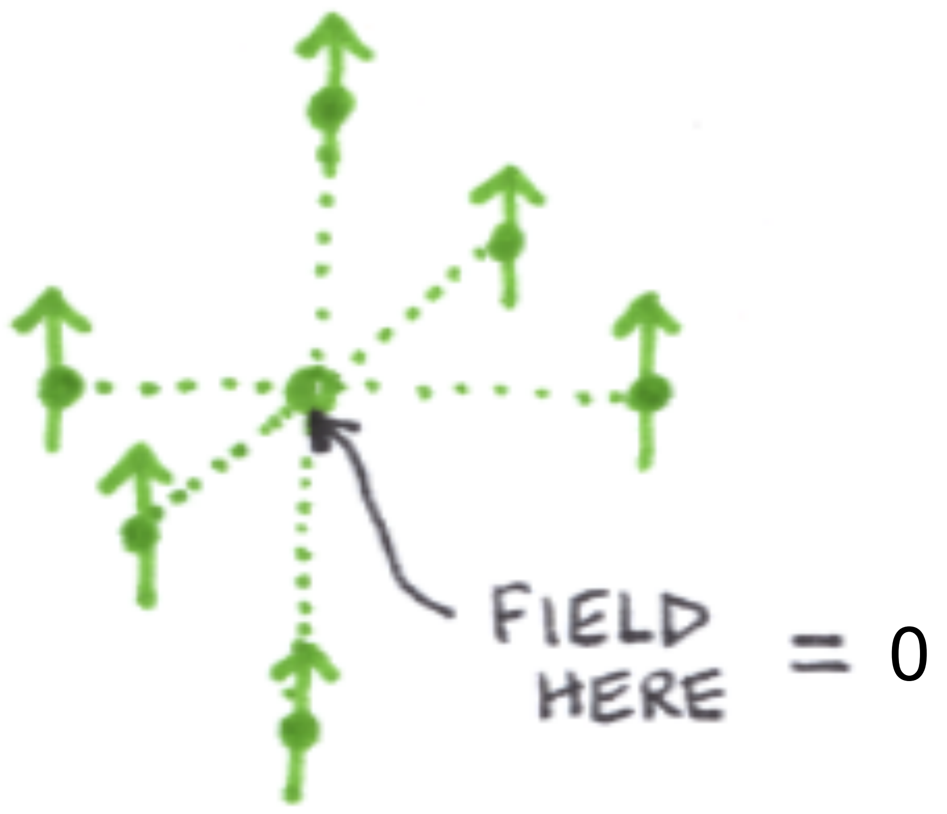
Dipole field: example 2, muon separated along *x*

$$\boldsymbol{D} = \frac{\mu_0}{4\pi}\begin{pmatrix} \frac{2}{x^3} & 0 & 0 \\ 0 & -\frac{1}{x^3} & 0 \\ 0 & 0 & -\frac{1}{x^3} \end{pmatrix}$$



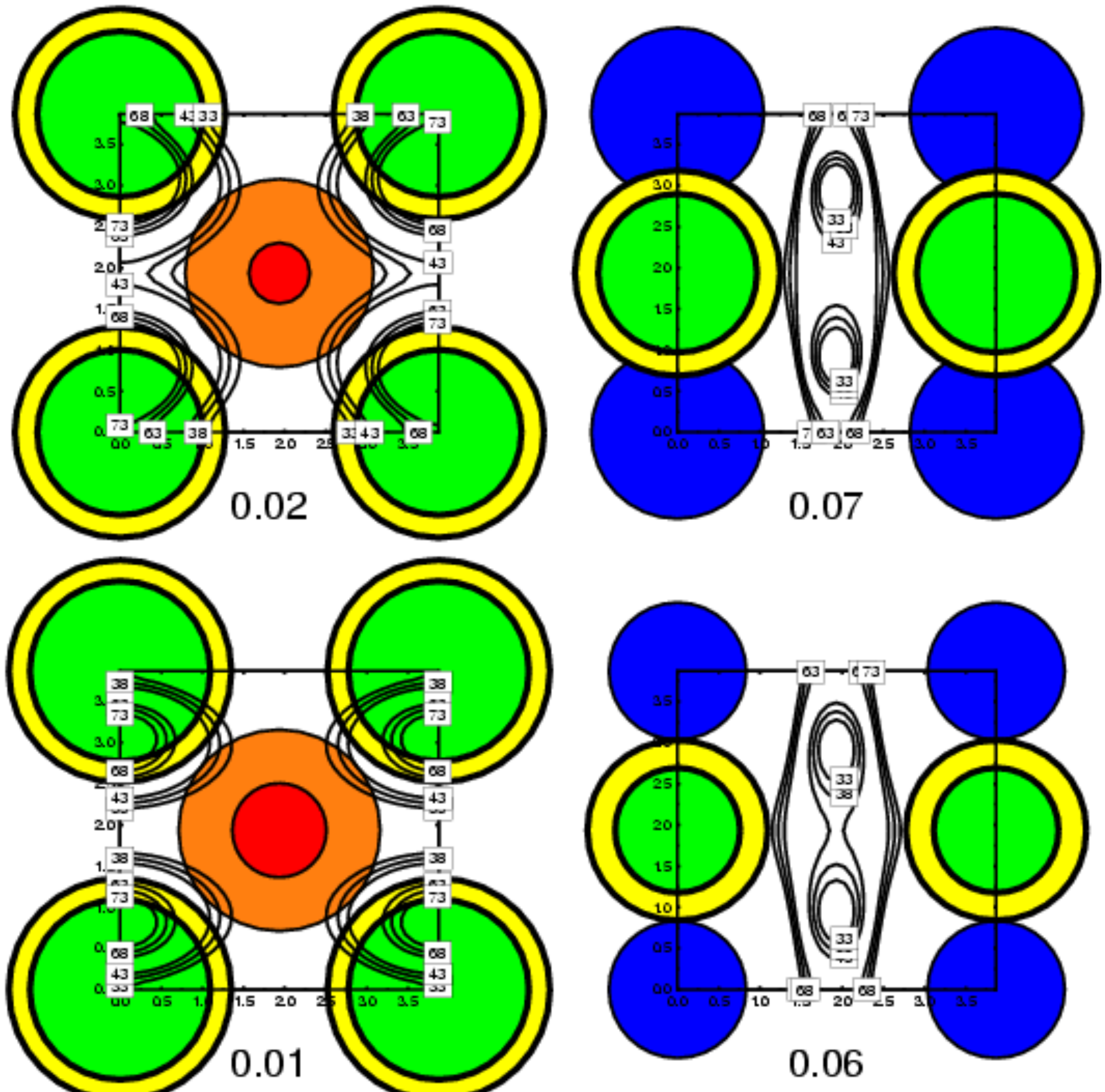
Dipole field: example 3

Problem:



Dipolar fields

Dipolar fields
can be calculated



Local magnetic field at the muon site

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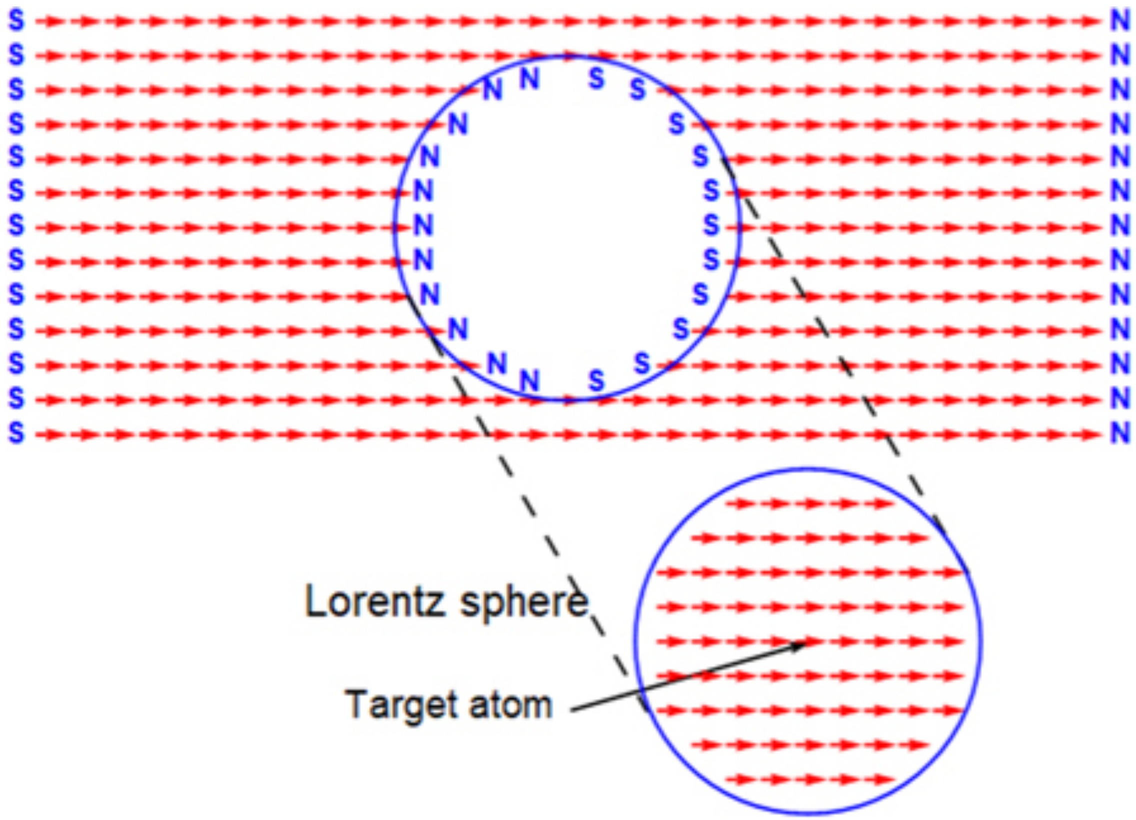
  due to electron spin density at muon site

* B_{demag} DEMAGNETIZATION FIELD

  depends on sample shape or domain structure

Lorentz field



Spins outside the sphere lead to an extra contribution $\boldsymbol{B} = \mu_0 \boldsymbol{M}/3$

Local magnetic field at the muon site

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  due to electron spin density at muon site

* B_{demag} **DEMAGNETIZATION FIELD**

  depends on sample shape or domain structure

Case study:
Incommensurate magnetic order

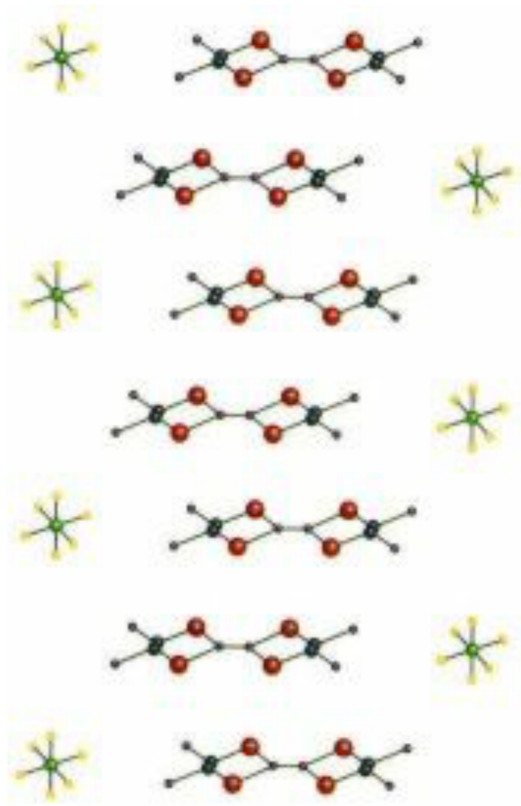
More on relaxation functions

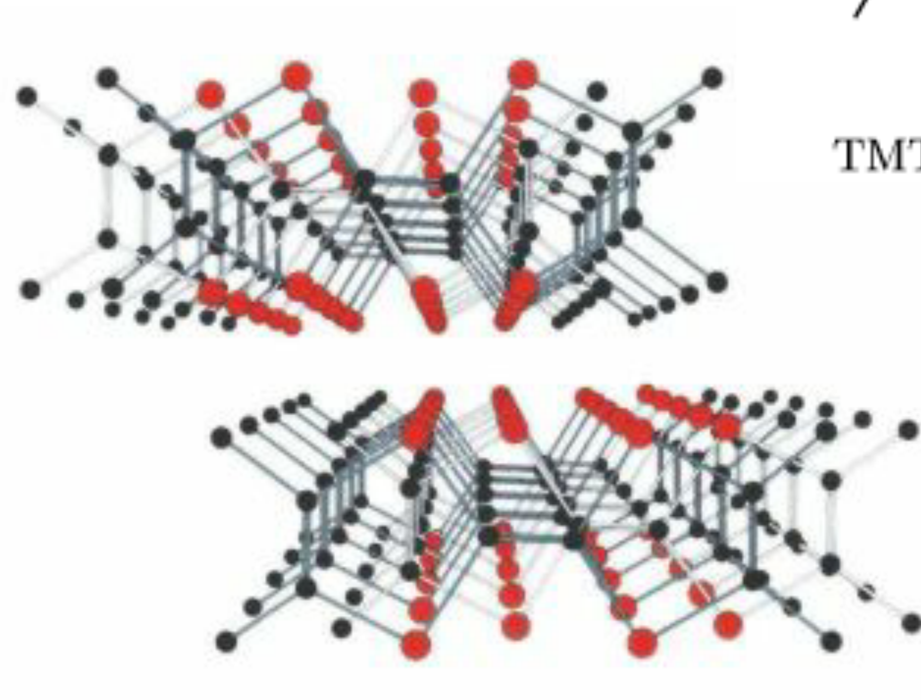
$$A(t) \sim \sum_i A_i \cos(\gamma_\mu |B_i| t)$$

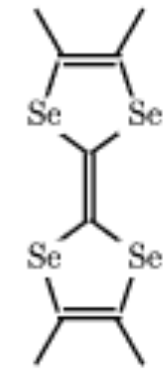
muon sites — field at site

In general $A(t) \sim \int p(B) \cos(\gamma_\mu B t) \mathrm{d}B$

TMTSF salts





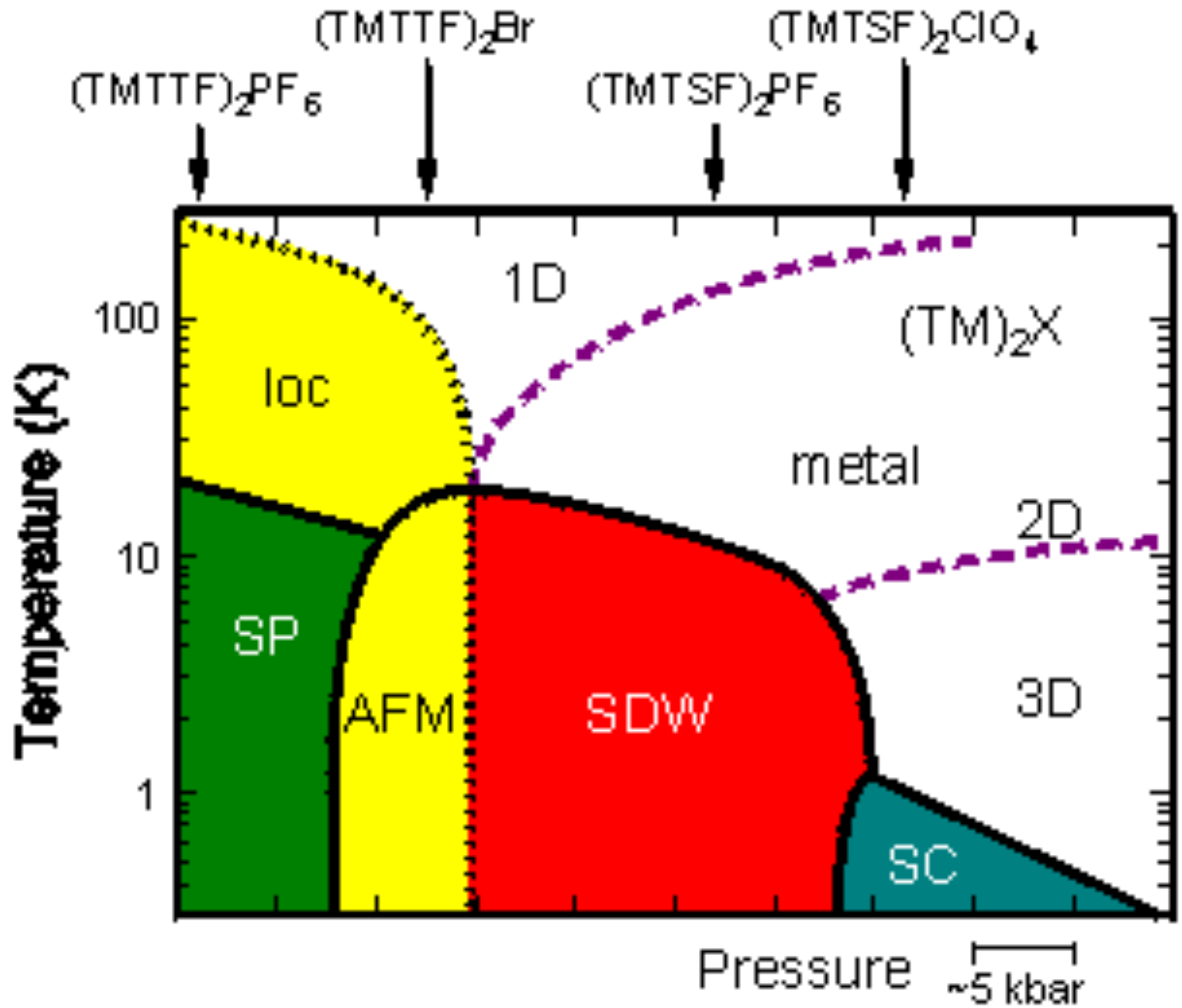


TMTSF

Stacks of TMTSF molecules $\Rightarrow$ 1D chains

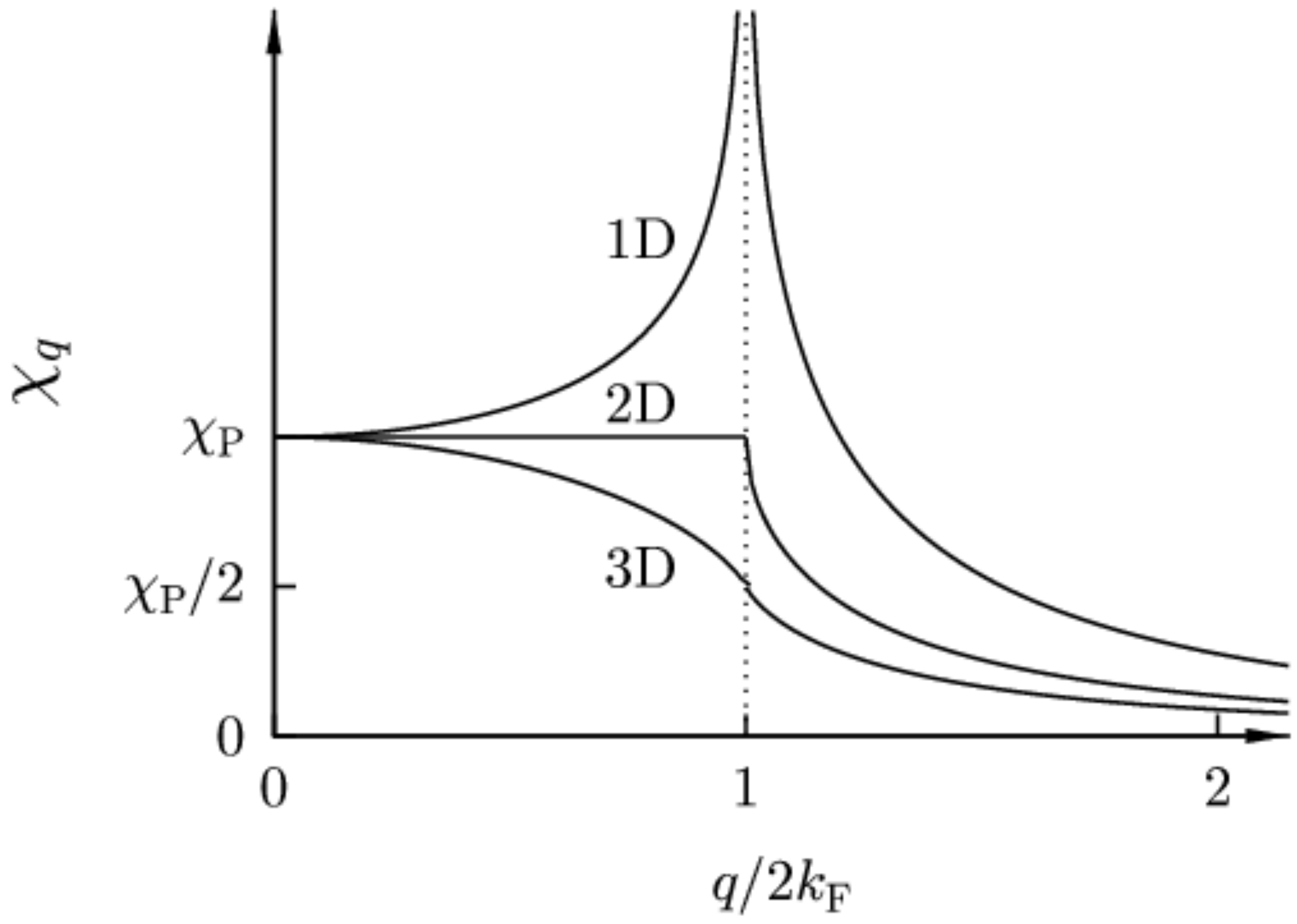
TMTSF salts

Very rich
phase diagram

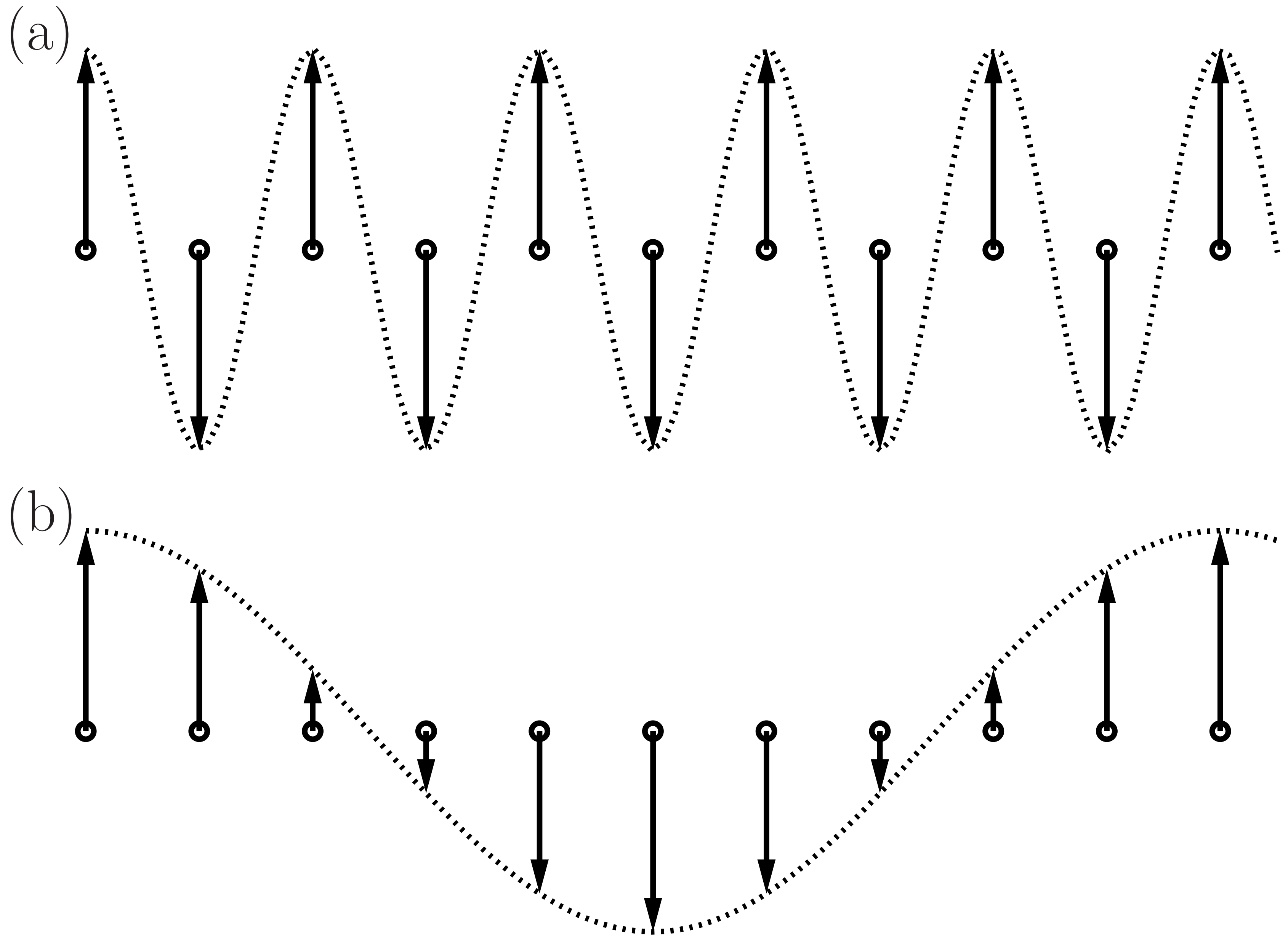


Paul Chaikin (1945 --)

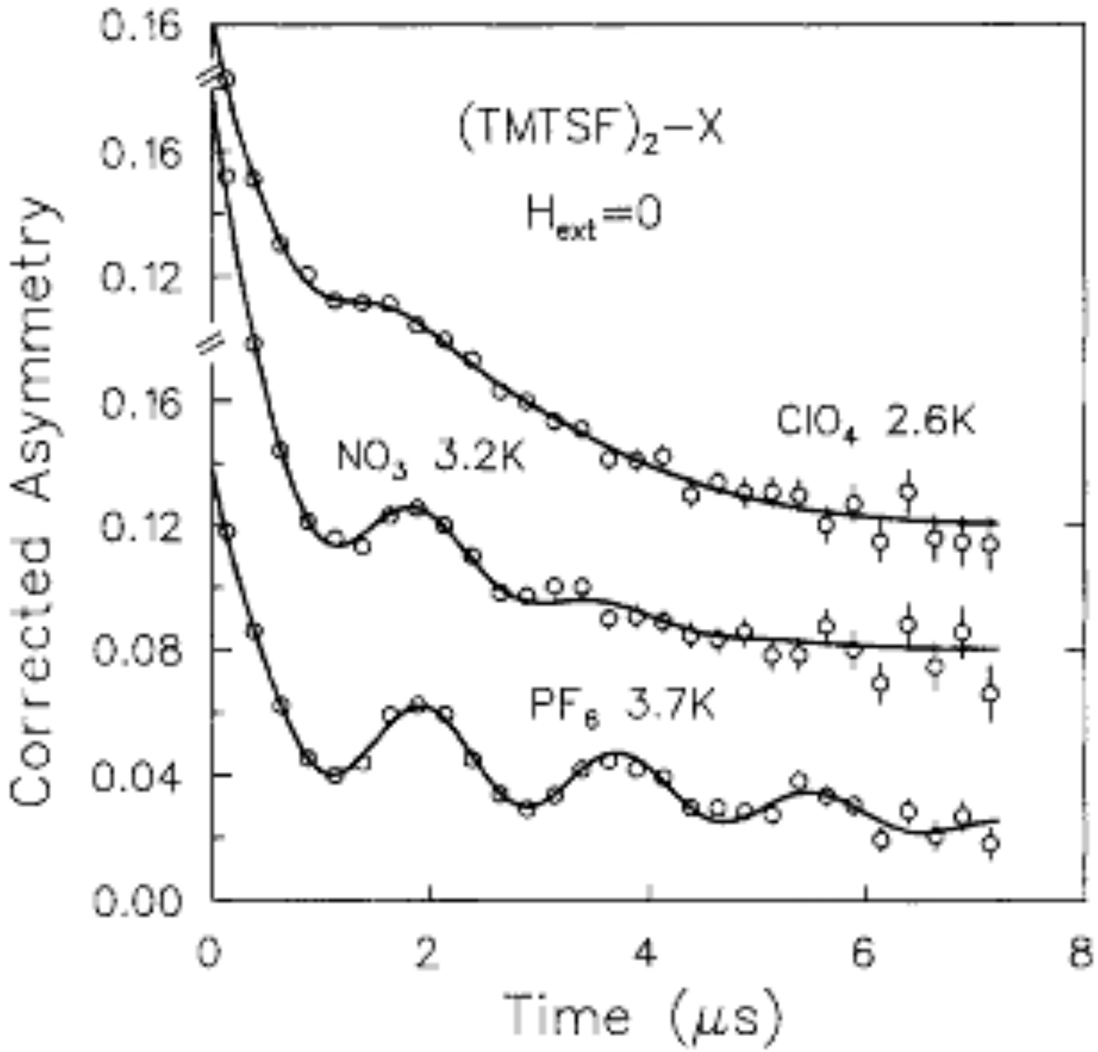
1D electron gas unstable to SDW formation



Spin-density wave

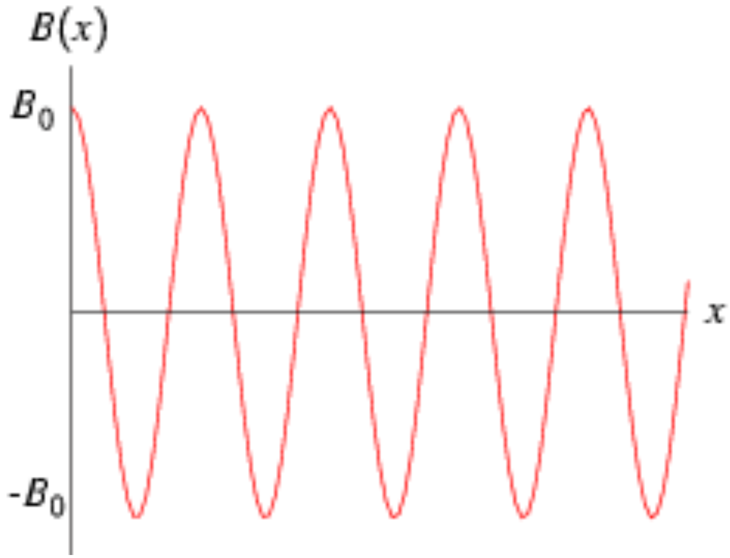


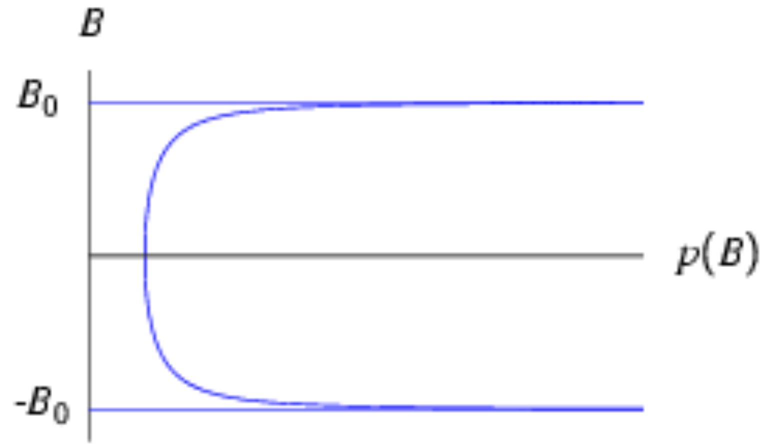
Muon data measured on TMTSF$_2$*X*



L.P. Le et al, PRB **48** 7284 (1993)

Spin density wave system: μ^+SR response





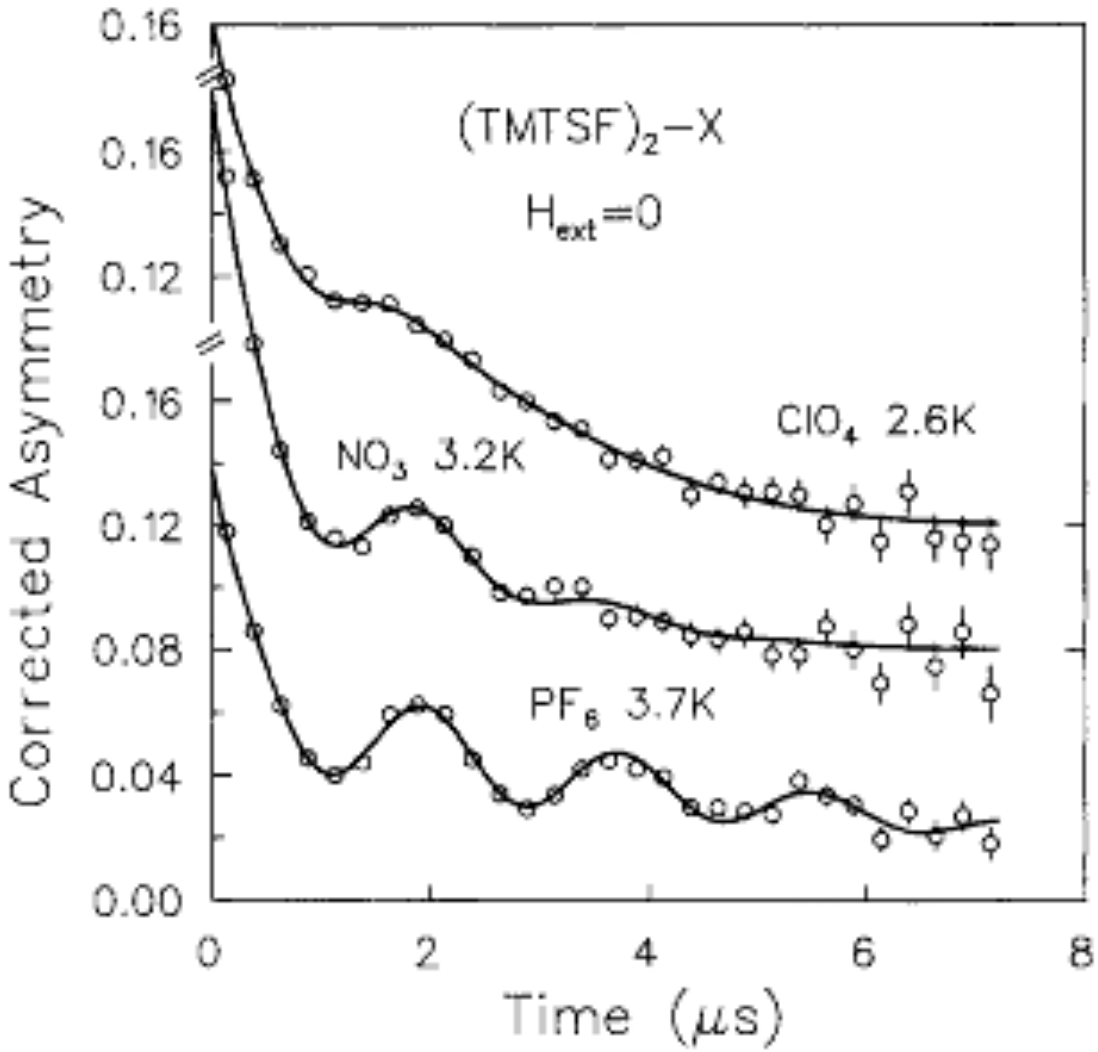
$$B(x) = B_0 \cos kx$$

$$p(B) = \frac{1}{\pi} \frac{1}{(B_0^2 - B^2)^{\frac{1}{2}}}$$

$$P_z(t) = \frac{1}{\pi} \int_{B=-B_0}^{B_0} \mathrm{d}B \, \frac{\cos(\gamma_\mu B t)}{(B_0^2 - B^2)^{\frac{1}{2}}} = J_0(\gamma_\mu B_0 t)$$

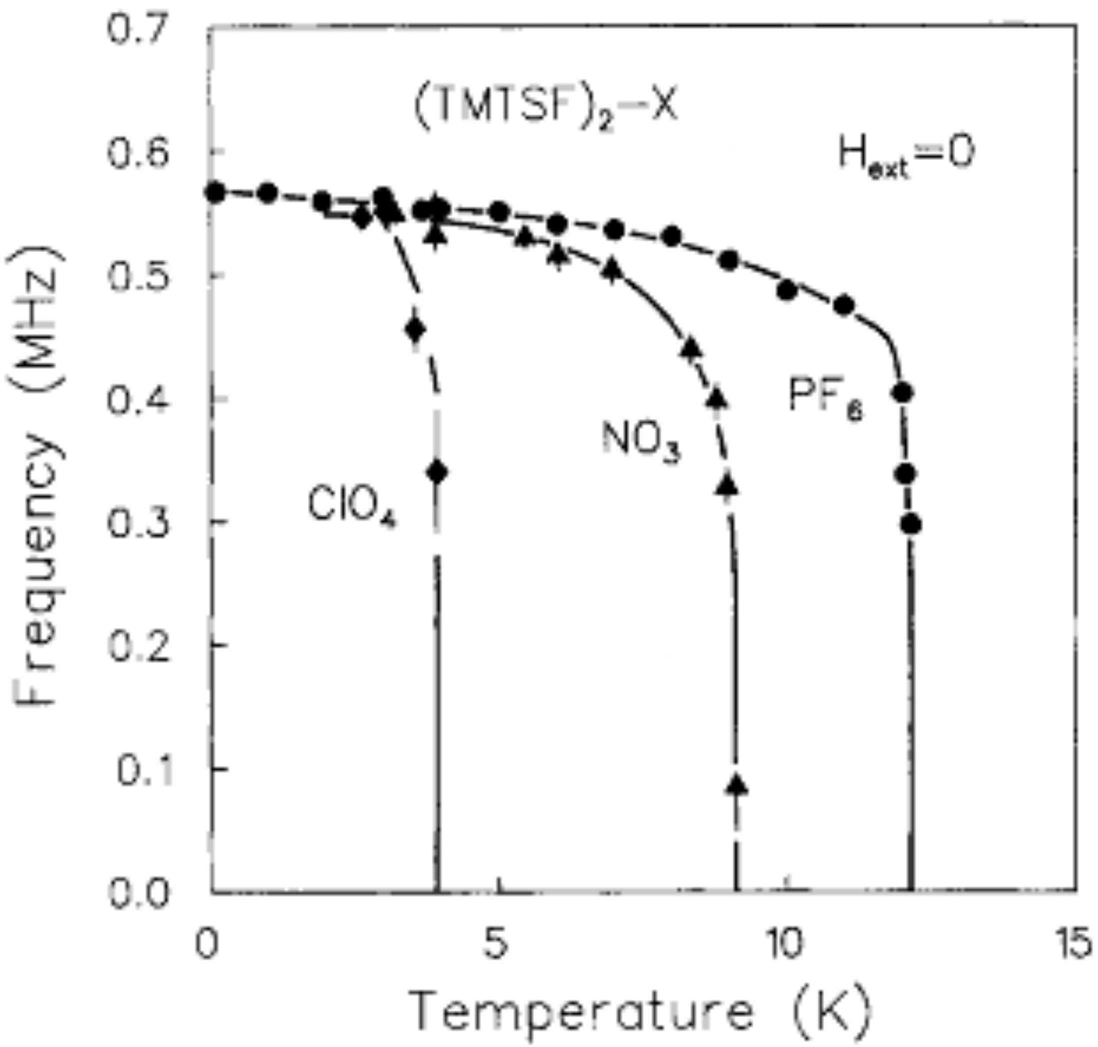
For large argument $J_0(\gamma_\mu B_0 t) \approx \left(\frac{2}{\pi \gamma_\mu B_0 t} \right) \cos\left(\gamma_\mu B t - \frac{\pi}{4} \right)$

Muon data measured on $TMTSF_2X$



L.P. Le et al, PRB **48** 7284 (1993)

SDW phase in $(TMTSF)_2X$



L.P. Le et al, PRB **48** 7284 (1993)

Summary

- Static magnetic order can lead to oscillations

- Oscillation frequency is an effective order parameter for the system

- We are sensitive to local, dipolar fields

- More complex field distributions lead to more complicated oscillatory spectra